Quantitative Reasoning and / or Quantitative Literacy

CSU Stanislaus Quantitative Reasoning Working Group

San Jose Meeting

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http://csustan.csustan.edu/qr

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Our Definition,
Expanded Definition,
and
Rubrics
of
Quantitative Reasoning

A General Mission Statement / Definition

Quantitative Literacy(QL) – also known as Quantitative Reasoning (QR) – is a “quantitative habit of mind”, proficiency, and comfort in dealing with and rationally processing numerical data. Individuals with strong QL skills possess the ability to analyze quantitative problems in everyday life situations using logical reasoning steps. They are able to read and understand numerical data. They can create valid arguments based on quantitative evidence and know how to interpret their conclusions. They are capable of clearly communicating their analyses and arguments in a variety of formats (including words, tables, graphs, mathematical equations and models, as appropriate).

Expanded definition:

The formal definition of Quantitative Literacy implies competency in different fields of basic mathematics, and their application to diverse problems in the sciences, business and administration, politics, economics, and in everyday life. Most importantly, QL requires an understanding of the mathematics that is deeper than mere memorization of, and facility with, calculation procedures. Possession of strong QL skills requires competency in critical areas:

1. Approximation / estimation – The ability to do effective approximation and estimation.
2. Mathematical models – The ability to understand the assumptions behind mathematical models, and the implications that those assumptions have for the validity and scope of conclusions that are drawn.
3. Tables and graphs – The ability to represent and understand data in graphical forms and other visual representations.
4. Algebra – The ability to understand and manipulate algebraic equations, including the ability to draw conclusions from functional dependencies. Competency in this area thus goes beyond the ability to substitute for known variables and to perform the requisite arithmetic.

5. Geometry – The ability to think and visualize in higher dimensions, including an understanding of the dependencies of geometric properties, such as volume, on the dimensions of the shapes. The ability to express properties in terms of angles.
6. Statistics – The ability to draw appropriate conclusions from statistical data, including an understanding of statistical distributions and properties such as average, median, and standard deviation. The ability to incorporate uncertainties in the data when drawing conclusions.

Rubrics

Assessment of Overall Quantitative Literacy (1)

Topic	Proficient	Adequate	Deficient
Logical Quantitative Reasoning and Analysis	Can understand problems and develop their own innovative logical quantitative analyses	Can follow and reproduce logical quantitative analyses	Analyses are based principally on random thoughts and guesswork

Assessment of Overall Quantitative Literacy (2)

Topic	Proficient	Adequate	Deficient
Validity, applicability, and limitations of quantitative arguments	Adept at developing valid quantitative arguments and under-standing their assumptions, applicability, and limitations	Can apply learned arguments to similar problems, but also tries to apply them to problems which are beyond the scope of the argument / analysis	Tries to solve all problems using the same strategies without adaptation.

Assessment of Specific Literacies

1. Approximation / Estimation

Proficient	Adequate	Deficient
Can effectively perform meaningful and novel estimates and approximations.	Can incorporate estimated data to estimate expected results.	Over reliance on calculators. Views all answers as precise. Unable to distinguish between accuracy and precision.

2. Mathematical / Quantitative Models

Proficient	Adequate	Deficient
Can analyze a real world example sufficiently to conceptualize a simulation of the system, which is complex enough to give meaningful results, but simple enough to be understandable.	Can simplify complex models to obtain approximate results and understand the limitations imposed by that approximation.	Can recognize and appreciate meaningful use of models to understand the world at large.

3. Tables and Graphs

Proficient	Adequate	Deficient
Can understand data presented in tabular or graphical form, and recognize trends in data. Understands the validity of extrapolation. Can develop sketch graphs.	Can extract relevant data from tables or graphs, and interpolate, but without ability to see the overall picture. Recognize importance of scales, labels, and error bars.	Can retrieve data from tables. Fails to recognize importance of scales, labels, and error bars.

4. Algebraic / Symbolic Approaches

Proficient	Adequate	Deficient
Understands functional relationships, can manipulate symbols, and can draw conclusions from each. Can effectively solve novel word problems.	Can reliably manipulate algebraic / symbolic expressions. Can handle more than one equation at a time. Can identify and apply similar methods to corresponding word problems.	Can only “plug and chug”. Makes frequent mistakes when manipulating equations. Cannot do word problems.

5. Geometry

Proficient	Adequate	Deficient
Can visualize in three dimensions. Can scale shapes. Effective at applying geometric methods to specific problems.	Can visualize above two dimensions only for simple shapes. Unable to scale shapes. Competent at using angular measurements in multiple dimensions.	Confusion about differences between volume and area. Difficulty with angular measurements.

6. Statistics and Probability

Proficient	Adequate	Deficient
Can interpret data in other ("non-normal") distributions. Understands the validity and limitations of sampling methods including the chance of false positives. Familiarity with probabilistic reasoning.	Can interpret data in a normal distribution, including concepts such as mean, mode, median, and standard deviation. Understands basic ideas of sampling methods and error analysis.	Understands simple average, but does not distinguish mean, mode, median. Is unfamiliar with standard deviation. No clear understanding of sampling methods and error analysis.

Curricular Modules:

We are developing various curricular modules in a variety of disciplines for integration into diverse upper division courses, including both major courses and upper division General Education courses.

Included here are brief overviews of several examples. Additional material is available here:

http://csustan.csustan.edu/qr

Example Curricular Modules

- Water
  - Spanish Literature class
  - Sociology Approach
- Estimation
  - Birth rate
  - Radioactive toxicity
- Universal Pre-School

Water

CSU Stanislaus Quantitative Reasoning with Water
Melanie Martin, Sandra Garcia-Sanborn, Jey Strangfeld
Computer Science, Modern Languages, Sociology

This module is designed for a Spanish Literature Class

Water -- A Story

- Read and analyze Costa Rican writer Carmen Naranjo's short story "Y vendimos la lluvia," published in 1989. English title: "And we sold the rain."
- The story satirizes the complexity and consequences of the economic dependency of developing countries.
- The core of the problem is that they have already exported all their primary resources: coffee, rice, sugar cane, vegetables, wood, etc., and the country and its people are still broke.

A Solution

- A middle-eastern Sultan hears of the land where rain abounds and a plan is hatched to sell the water to the Sultan for 10 dollars per cubic centimeter.
- The consequences are economically and environmentally devastating for this developing country and its people, as they lose their main natural resources (due to the severe drought).
- How can we understand 10 dollars per cubic centimeter as a price for water?

Activities

- **First Activity**: Have students make a square centimeter with a ruler to start to visualize the units in the story.
  - Discuss area and volume
  - Practice conversions
    - Inches
    - Meters
    - …

Activities

- **Second Activity:** To get a notion of the difference between the level of rain in a tropical country in comparison with a non-tropical region, find out which is the average annual rainfall in Costa Rica, which is around 100 inches per year (with up to 25 feet in some regions), and in the State of California OR in the Central Valley.
  - Web search
  - Graphing of data
  - Scale

Activities

- **Third Activity:** Have students choose 3 personal activities (from a given list of five) which involve the use of water at home and calculate their daily, monthly, and yearly individual use.
- They may need to measure the flow per time unit for their faucet and then measure how long they have the water running to estimate their water useage.
- **Choose 3 activities out of 5 given to calculate water usage**
  - Shower
  - Teeth brushing
  - Hand washing
  - Toilet flushing
  - Dishwashing

Activities

- Activity 3 Continued
  - Students can bring their data to class and enter it into a Google Doc spreadsheet
  - Compute descriptive statistics for the class data
  - Raise questions
  - Make hypotheses
  - Discuss how to test them

Activities

- **Fourth Activity:** Calculate how much you waste when comparing with the average use. Calculate how much it would cost you per day, month, year, if you would have to buy that water from Costa Rica at the given price in the story. Working in groups according to matching "domestic" activities, have students compare their uses and expenses.

Water – a Sociology Focus

- Water Footprint Assessment
- Personal Water Footprint Calculator
- http://waterfootprint.org/en/resources/interactive-tools/personal-water-footprint-calculator/
- Footprint Calculator – the water footprint calculator in its most simplistic and basic form asks respondents to answer the following questions
  - Country of residence
  - Gender (closed-ended, male/female)
  - Individual portion of family income (open-ended, in dollars)
  - Vegetarian or meat consumer (closed-ended, vegetarian, average, and high meat consumption. “Average” and “High” are subjective)

Sociology

- Personal water footprint calculated from answers above. Compares their footprint to the global average. It also breaks their footprint in terms of the impact of different food categories (i.e. meat, dairy, fruits and vegetables, etc). Student can write down their calculated water footprint and use this information to do different activities focusing on building quantitative reasoning.

Sociology

- First activity
  - Can plot class members on a simple graph. Can look for estimates of averages and ranges on graph. Can also use global average (1243 m3 per year) as a comparison point.
  - Can calculate descriptive statistics (i.e. median, mean, range, and standard deviation)
  - Critical analysis. What does this information tell is about our water consumption? What other important variables are not included?

Sociology

- Second activity
  - Ask students to change their meat consumption choice. Record their footprint calculator. Ask them to change their gender. Ask them to change their income by increasing in by $10k and then decreasing in by $10k. Record all of their findings.
  - Have students compare their modified scores to their original score
    - What impact does gender have on water footprint and why?
    - What impact does meat consumption have on water footprint and why?
    - What impact does income (class) have on water footprint and why?
  - Critical analysis: Have students read articles about the impact of class and consumption on water footprint and incorporate into their analysis.

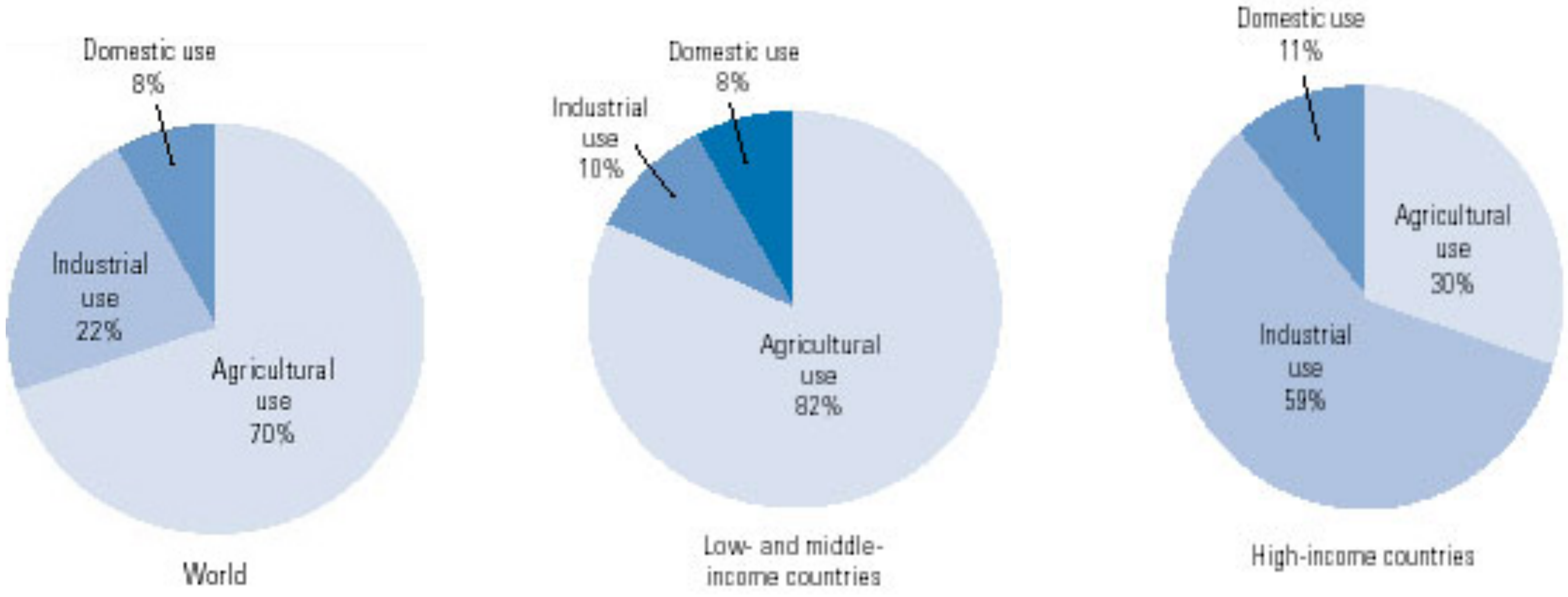
Pre-Assessment

- Understanding the impact of class and wealth on water use.
- Testing: Rubric 3: Understanding Tables and Graphs (Questions 1-5)

  Rubric 1: Estimation (Questions 4-5)

  Rubric 4: Algebraic/Symbolic Approaches (Question 3)
- The pie-charts below indicate water usage by country income level and sector usage. Compare water use information in the three charts. What is water used for primarily in terms of world global consumption, consumption in low and middle income countries, and consumption in high income countries?

Competing water uses for groups of countries

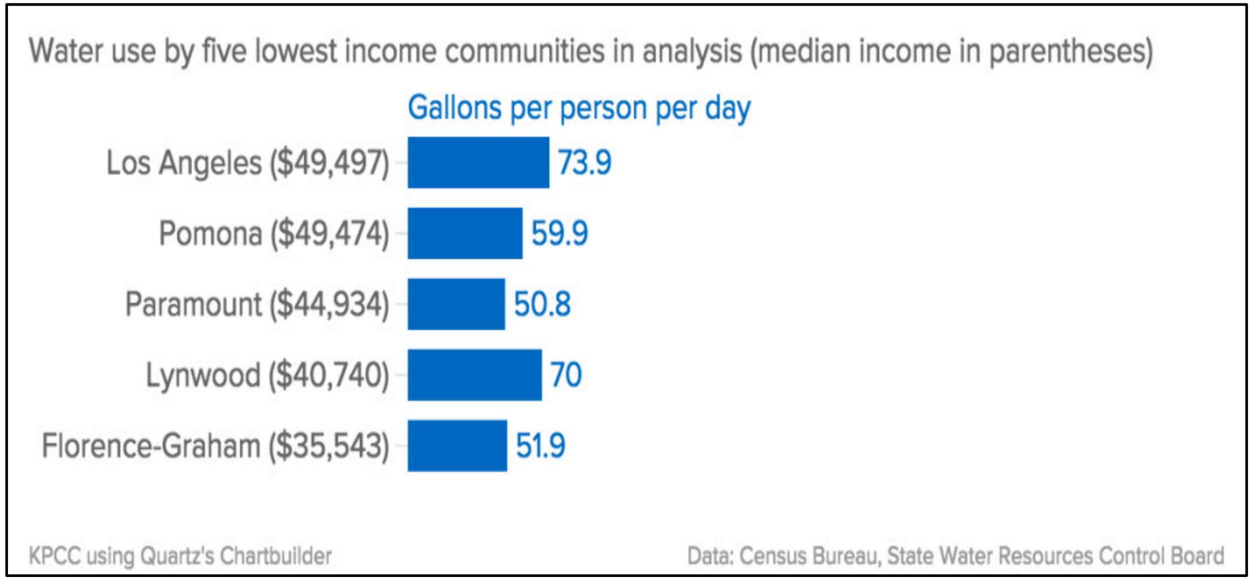


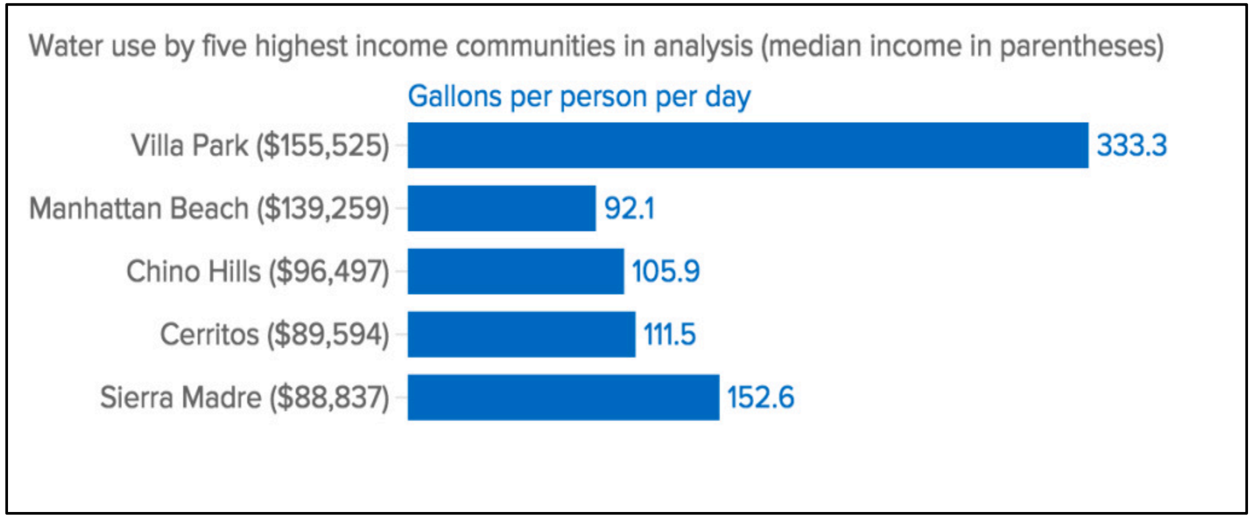
Extracted from the Executive summary of the WWDR. World Bank, 2001. . Washington DC.

- The table below also indicates water usage by country income level and sector usage.
  - Is the pattern of sector usage (agricultural, industrial, and domestic) similar to the data posted above across income categories? Explain your answer.
  - Compare the countries groups in terms of their overall water withdrawal. What pattern do you see?
  - What general information about differences in water usage does this table give you that is missing in the above pie charts?

Country income group	Annual withdrawals per capita	Withdrawals by sector		
		Agric.	Ind.	Dom.
	(...m^3...)	(...............%.........)		
Low-income	386	91	5	4
Middle-income	453	69	18	13
High-income	1 167	39	47	14

- The chart below looks at variations in per capita water usage in 10 different counties in Southern California.
  - What are the patterns you see here and are they similar or different from what you would given the information in the charts above?
  - What does the concept of median income mean?
  - If $1m^2$ = 264.172 gallons, how many gallons per day you would use based on your personal water footprint estimator.
  - Is your personal water footprint in gallons per day similar to what you would expect given the data below.


Water use by five lowest income communities in analysis (median income in parentheses)
Gallons per person per day
Los Angeles ($49,497)
73.9
Pomona ($49,474)
59.9
Paramount ($44,934)
50.8
Lynwood ($40,740)
70
Florence-Graham ($35,543)
51.9
KPCC using Quartz's Chartbuilder
Data: Census Bureau, State Water Resources Control Board


Water use by five highest income communities in analysis (median income in parentheses)
Gallons per person per day
Villa Park ($155,525)
333.3
Manhattan Beach ($139,259)
92.1
Chino Hills ($96,497)
105.9
Cerritos ($89,594)
111.5
Sierra Madre ($88,837)
152.6

Estimation

This module is being used in upper division General Education, paired courses between English and Physics.

Introduction

- This exercise was meant to develop a class estimate of the amount of radioactive material which was given to Alexander Litvinenko, a Russian dissident living in exile in London. All the required physics has been covered already in class, after which there are two remaining problems
- There are a number of steps involved, and for non-science majors to navigate their way through them is asking a lot.
- Although some quantities are known (principally from the properties of the radioactive isotope) others are not and reasonable assumptions have to be made.
- We start with an unrelated example to introduce the idea of estimation before tackling the problem at hand.

Estimation 1

- Question: How many babies will be born in the US in the next hour?
- Assumptions:
- The population of the US is constant. If that is true then the number of babies born must
- equal the number of deaths. Not strictly true but then a. the rate of change is small, and this is only an estimation b. some of the change is due to a net immigration into the US
- Population of the US is about 300 million
- Average human lifespan in the US is about 80 years
- Birth rate is constant day by day, and hour by hour.

Estimation:

1. Number of deaths per year = 300 million / 80 = 3,750,000 per year
2. Number of deaths per day = 3,750,000 / 365 days = 10,000 per day
3. Number of deaths per hour = 10,000 / 24 hours = 400 per hour
4. Number of births per hour = 400 per hour

Notes:

1. Actual number of births in 2013 = 3,932,181
   http://www.cdc.gov/nchs/births.htm
2. Assuming a constant rate throughout the year this is a rate of 450 per hour.

Estimation 2

- Question: How much radioactive material was given to Alexander Litvinenko?
- Known:
  1. Poison was ^{210}Po.
  2. ^{210}Po is an α emitter with an energy of 5.4 MeV (8.6×10^{-13} J).
  3. Atomic mass of ^{210}Po = 209.98 amu = 3.5×10^{-25} kg.
  4. Half life of ^{210}Po = 138 days = 1.2×10^{7} s. 5. From this the decay constant = ln2/1.2×10^{7} s = 5.8×10^{-8} per second.

Assumptions:

1. 100 % of α particle energy was absorbed.
2. Quality factor (rems per rad) for α particles is 25
3. Mass of victim = 100 kg
4. Dose equals LD50 value of 450 rem.

- Students were given the flow chart in figure 1, except all boxes were left blank, to be filled in by them. After filling all knowns and assumed values the chart shows a clear path from the last assumption (the dose) to the amount of material given to the victim.
- Known quantities are filled in first (shown in red) followed by the assumed quantities (shown in blue). Once these have been entered there is a path leading up the page from the assumed dose (450 rem) to the final answer for the amount of material (1.5 μg)

Figure 1a: Flow diagram outlining variables in each box, and relationships between them.

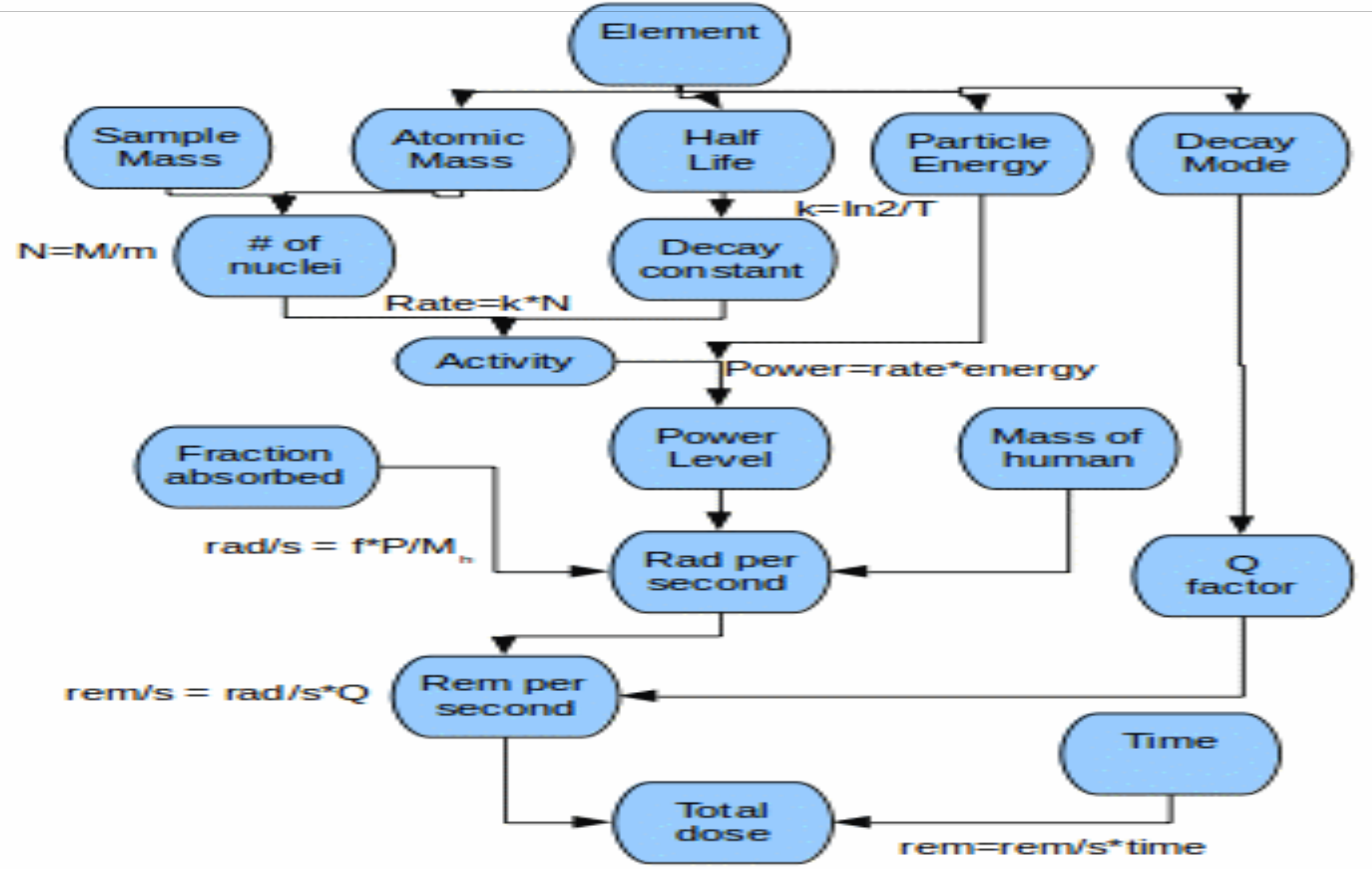
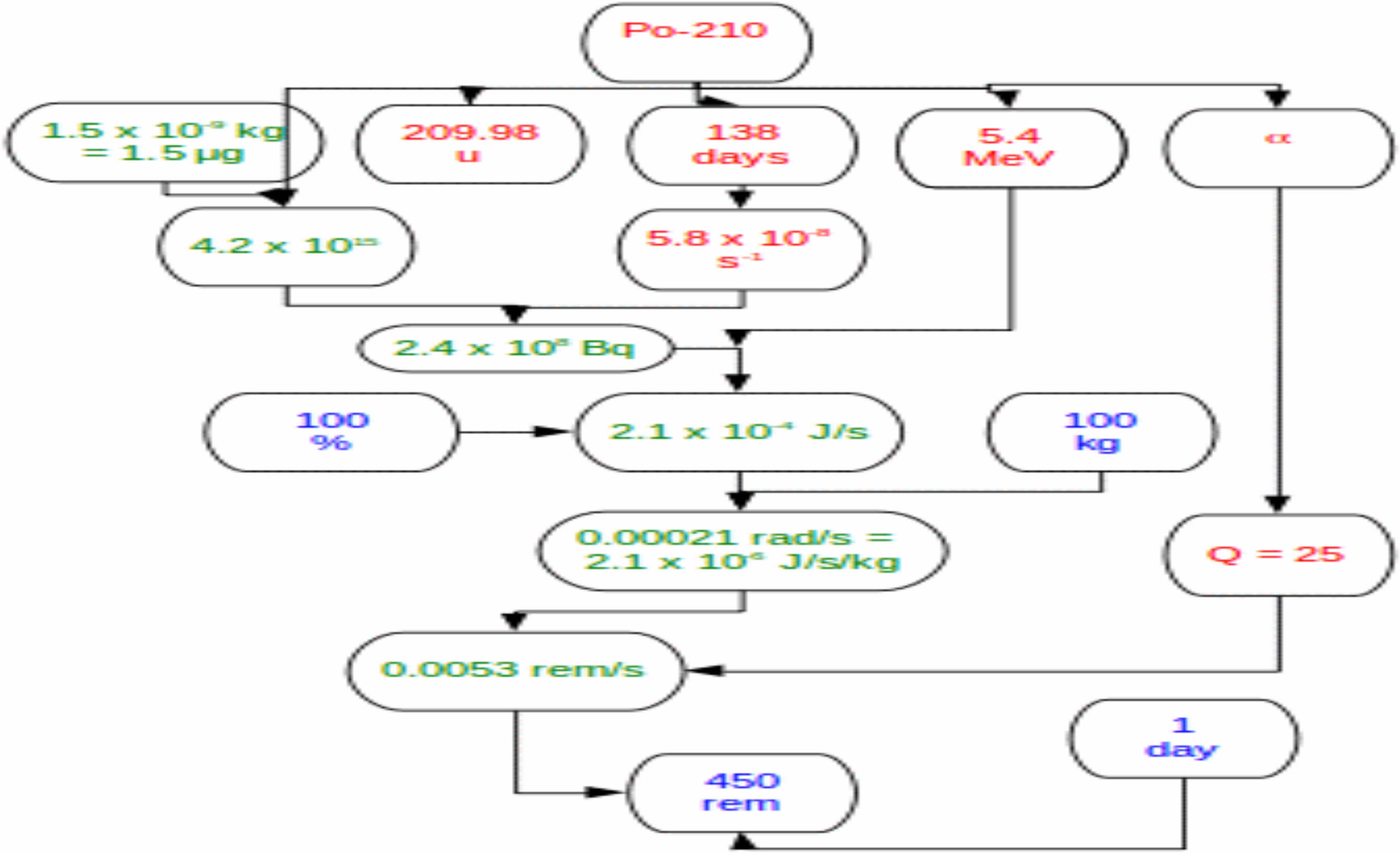


Figure 1b: Assignment of variables. Known variables are in red, assumed variables in blue, and calculated variables in green.



Discussion

- The resulting inquiry found that Mr Litvinenko ingested about 10 μg(2) of ^{210}Po, somewhat larger than our estimated value. However, our estimate is within an order of magnitude of the accepted value, a good result given the simplicity of the calculation. Furthermore it is easy to see the principal reason for the difference. We have assumed that he was given the LD50 dose, that is 450 rem, whereas any assassin would surely give a much larger dose than that likely to have a 50% chance of killing his victim. If we increase our assumed dose from 450 rem to about 2500 rem, then our final answer for the amount of ^{210}Po would be in very good agreement with the accepted value.

Should the State assure universal pre-school opportunities for all children?

A Quantitative Reasoning Approach

Melanie Martin

Should the State assure universal pre-school opportunities for all children?

- What is the question?
- Definitions
  - State
  - Pre-School
  - All Children

Definition

- Assume state of California (US?)
  - Would this be through school districts or stand alone?
  - Who would oversee?
  - Where would it be held?
  - What hours?
  - Interface with Head Start or other programs?

Definitions

- What is pre-school – content, timing
  - When does it move from daycare to school?
  - Qualifications of teachers?
  - Whole day, half day, shorter?
- What age of children?
  - 2-5 yrs?
  - 3-5 yrs?
  - 4-5 yrs?

Stepping Back

- Why are we asking this?
- Is there a problem?
- If there is a problem what is the scale?
- What is the interest of the state in this?
- How would we make a decision?
  - Money
    - Cheaper than prison later
  - Educational outcomes
    - More likely to finish high school, college

Evaluate my Biases

- Do I care or have an opinion on this issue?
- How can I avoid having it affect my data selection and evaluation?

What Data Do I Need?

- What is the current situation:
  - when do kids start school in CA,
  - is Transitional Kindergarten universally available,
  - what preschool programs are available
  - Are available programs effective?

What Data Do I Need?

- Has anyone tried it?
  - Credible studies
  - Outcomes
  - Similarity to our situation
    - Does it apply

What Data Do I Need?

- California High Schools
  - Graduation and drop-out rates broken down by type of pre-K education
  - Outcomes for high school graduates broken down by type of pre-K education
  - ……

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