The Logistic Flow

(continuous)

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Complex Systems Summer School

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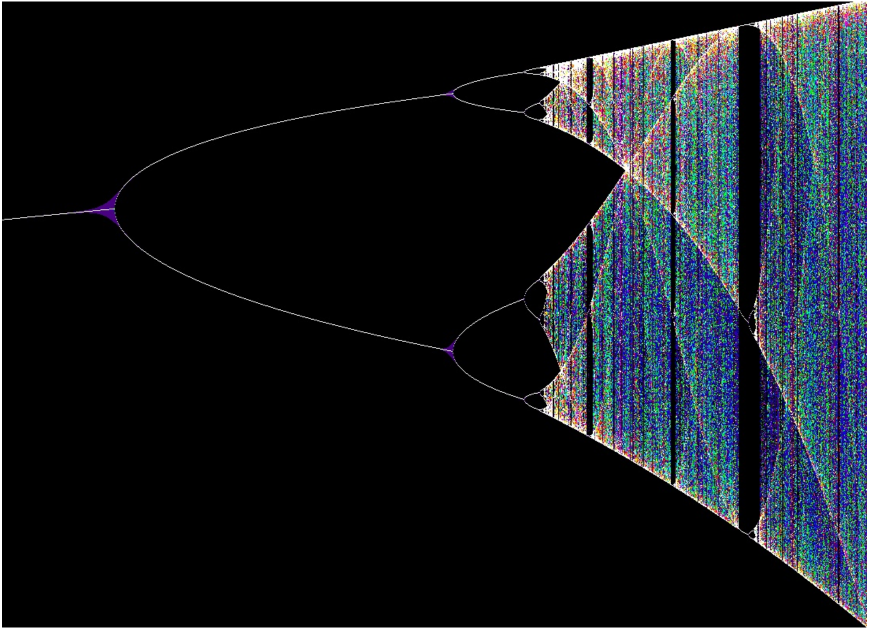
Discrete logistic map

We all know that the discrete logistic map

$$P_{n+1} = rP_n(1 - P_n)$$

exhibits interesting behavior of various sorts for various values of the parameter r, including chaos, etc.

Discrete logistic map – bifurcation diagram



Continuous logistic flow

What kind of behavior can we expect from a continuous version of a logistic flow:

$$\frac{dP}{dt} = rP(1-P) \quad ?$$

Continuous logistic flow - solving

Note that this is a non-linear ODE, but fortunately we can actually integrate …

$$\begin{aligned} \frac{dP}{dt} &= rP(1-P) \\ \frac{dP}{P(1-P)} &= rdt \end{aligned}$$

Thus:

$$\begin{aligned} \int \frac{dP}{P(1-P)} &= \int rdt \\ \int \frac{dP}{P(1-P)} &= rt + c_1 \end{aligned}$$

Continuous logistic flow - solving

By partial fractions, we have:

$$\int \frac{dP}{P} + \int \frac{dP}{(1-P)} = rt + c_1$$

$$\log(P) - \log(1-P) = rt + c_1$$

$$\log\left(\frac{P}{1-P}\right) = rt + c_1$$

$$\frac{P}{1-P} = e^{rt+c_1}$$

$$\frac{P}{1-P} = c_2 e^{rt}$$

Continuous logistic flow - solving

This gives us:

$$P = (1 - P)c_2 e^{rt}$$

And thus:

$$\begin{aligned} P &= c_2 e^{rt} - P c_2 e^{rt} \\ P + P c_2 e^{rt} &= c_2 e^{rt} \\ P(1 + c_2 e^{rt}) &= c_2 e^{rt} \end{aligned}$$

Continuous logistic flow - solving

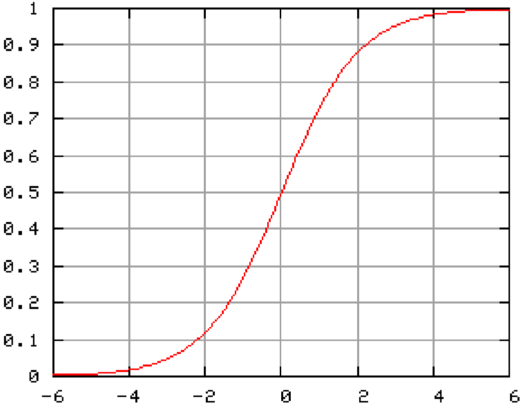
From this we get:

$$P = \frac{c_2 e^{rt}}{1 + c_2 e^{rt}}$$

Finally, dividing top and bottom by $c_2 e^{rt}$ and simplifying, we have:

$$P = \frac{1}{1 + ce^{-rt}}$$

The classic logistic/sigmoid curve



and changes in *c* and *r* make minor changes in the behavior near 0 . . .

Discrete vs. Continuous

The difference between the behavior of the discrete and continuous logistic functions can give us some idea of the significance of working in the discrete regime . . .

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Slides for this talk will be available at:

http://csustan.csustan.edu/~tom/SFI-CSSS/2009

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