The Logistic Flow

(Continuous)

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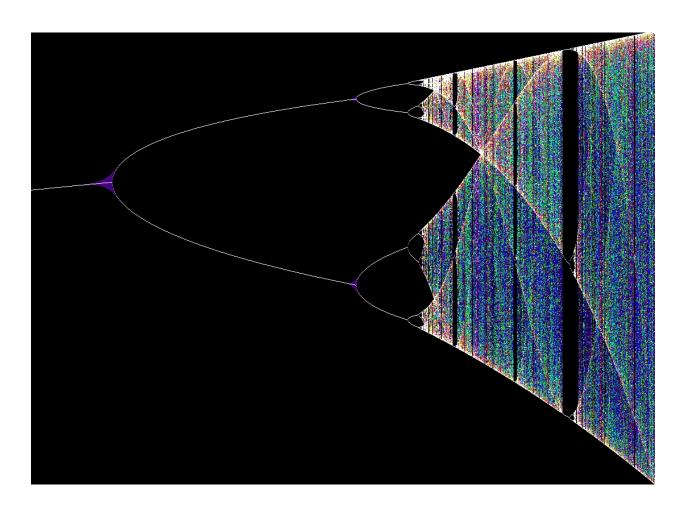
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Logistic flow . . . ←

We all know that the discrete logistic map

$$P_{n+1} = rP_n(1 - P_n)$$

exhibits interesting behavior of various sorts for various values of the parameter r, including chaos, etc.



What kind of behavior can we expect for the continuous version of a logistic flow:

$$\frac{dP}{dt} = rP(1-P) ?$$

Note that this is a non-linear ODE, but fortunately we can actually integrate . . .

$$\frac{dP}{dt} = rP(1-P)$$

$$\frac{dP}{P(1-P)} = rdt$$

Thus:

$$\int \frac{dP}{P(1-P)} = \int rdt$$

$$\int \frac{dP}{P(1-P)} = rt + c_1$$

By partial fractions, we have:

$$\int \frac{dP}{P} + \int \frac{dP}{(1-P)} = rt + c_1$$

$$\log(P) - \log(1-P) = rt + c_1$$

$$\log(\frac{P}{1-P}) = rt + c_1$$

$$\frac{P}{1-P} = e^{rt+c_1}$$

$$\frac{P}{1-P} = c_2 e^{rt}$$

$$P = (1-P)c_2 e^{rt}$$

And thus:

$$P = (1 - P)c_2e^{rt}$$

$$P = c_2e^{rt} - Pc_2e^{rt}$$

$$P + Pc_2e^{rt} = c_2e^{rt}$$

$$P(1 + c_2e^{rt}) = c_2e^{rt}$$

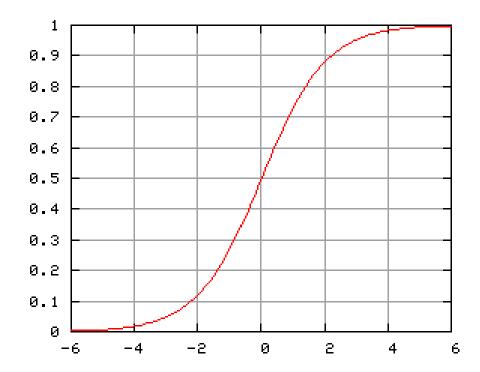
giving us:

$$P = \frac{c_2 e^{rt}}{1 + c_2 e^{rt}}$$

and, dividing top and bottom by c_2e^{rt} , and simplifying, we have:

$$P = \frac{1}{1 + ce^{-rt}}$$

This function just gives us the classic logistic/sigmoid curve:



and changes in c and r make minor changes in the behavior near 0 . . .

The difference between the behavior of the discrete and continuous logistic functions can give us some idea of the significance of working in the discrete regime . . .