N-K models
(boolean networks)

Tom Carter

Complex Systems Summer School

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Discrete logistic map

We all know that the discrete logistic map

$$P_{n+1} = rP_n(1 - P_n)$$

exhibits interesting behavior of various sorts for various values of the parameter r, including chaos, etc.

What kind of behavior can we expect from a continuous version of a logistic flow:

$$\frac{dP}{dt} = rP(1 - P) \quad ?$$

Continuous logistic flow - solving

Note that this is a non-linear ODE, but fortunately we can actually integrate . . .

$$\frac{dP}{dt} = rP(1 - P)$$

$$\frac{dP}{P(1 - P)} = r\,dt$$

Thus:

$$\int \frac{dP}{P(1 - P)} = \int r\,dt$$

$$\int \frac{dP}{P(1 - P)} = rt + c_1$$

Continuous logistic flow - solving

By partial fractions, we have:

$$\int \frac{dP}{P} + \int \frac{dP}{(1-P)} = rt + c_1$$

$$\log(P) - \log(1-P) = rt + c_1$$

$$\log(\frac{P}{1-P}) = rt + c_1$$

$$\frac{P}{1-P} = e^{rt+c_1}$$

$$\frac{P}{1-P} = c_2 e^{rt}$$

This gives us:

$$P = (1 - P)c_2 e^{rt}$$

And thus:

$$P = c_2 e^{rt} - Pc_2 e^{rt}$$

$$P + Pc_2 e^{rt} = c_2 e^{rt}$$

$$P(1 + c_2 e^{rt}) = c_2 e^{rt}$$

From this we get:

$$P \;=\; \frac{c_2 e^{rt}}{1 + c_2 e^{rt}}$$

Finally, dividing top and bottom by $c_2 e^{rt}$ and simplifying, we have:

$$P \;=\; \frac{1}{1 + c e^{-rt}}$$

The difference between the behavior of the discrete and continuous logistic functions can give us some idea of the significance of working in the discrete regime . . .

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Slides for this talk will be available at:
`http://csustan.csustan.edu/~tom/SFI-CSSS/2009`

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