Newton's Method

(and the square root)

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Newton's Method . . . $\leftarrow$

Newton's method (also sometimes known as the Newton-Raphson method) is a way to approximate the root(s) of a differentiable real-valued function.

The basic method is iterative. We start with an initial guess of a root, say, for example,

$$x_0 = 1$$

We then iterate (until we get close enough to a nearby root, we hope . . . :-)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

An example . . . finding the square root of a number a:

$$f(x) = x^2 - a$$

We then are using:

$$\begin{aligned} x_{n+1} &= x_n - \frac{x_n^2 - a}{2xn} \\ &= \frac{2x_n^2 - x_n^2 + a}{2x_n} \\ &= \frac{x_n^2 + a}{2x_n} \\ &= \frac{x_n + \frac{a}{x_n}}{2} \\ &= \text{Avg}(x_n, \frac{a}{x_n}) \end{aligned}$$

The ideas behind Newton's method:

We assume that $f(x)$ is differentiable. We are looking for a root of $f(x)$ (i.e., a place where $f(x) = 0$). Suppose that our current estimate of a root is x_n. We find the line tangent to $f(x)$ at x_n:

$$y = f'(x_n)(x - x_n) + f(x_n)$$

We find the place where this line crosses the x-axis, and let that value of x be our next estimate of the root we are looking for:

$$0 = f'(x_n)(x_{n+1} - x_n) + f(x_n)$$

Solving this for x_{n+1}, we get:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

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