Chaos You Can Play In

May Tan Lim, Erin Miller, Nicky Grigg, Aaron Clauset

SFI Complex System Summer School

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Outline

- *Experimental setup*
- *Equations of Motion*
- *The Lorentz Equations*
- *Mathematical Simulation*
- *Data Analysis*
- *Getting Lucky*

Experimental Setup

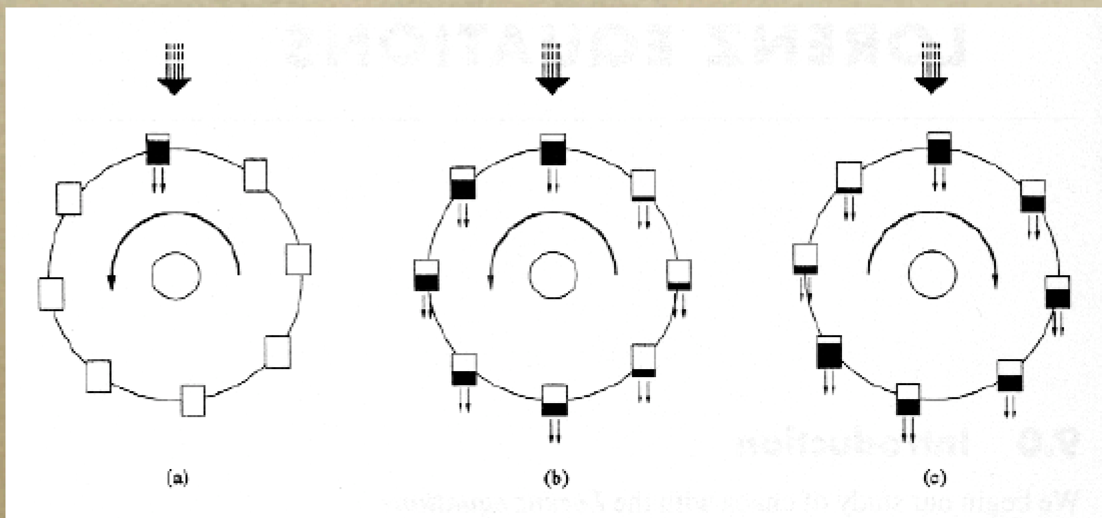
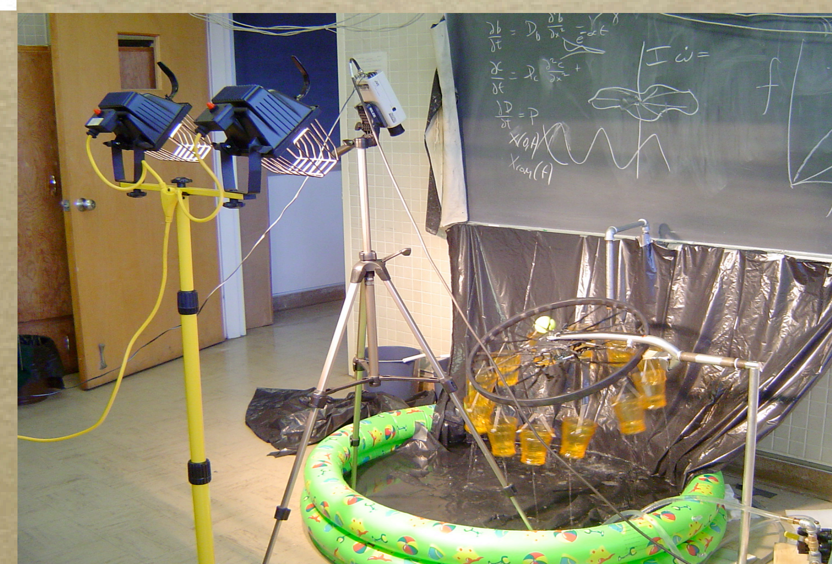


Diagram from Strogatz (1994)

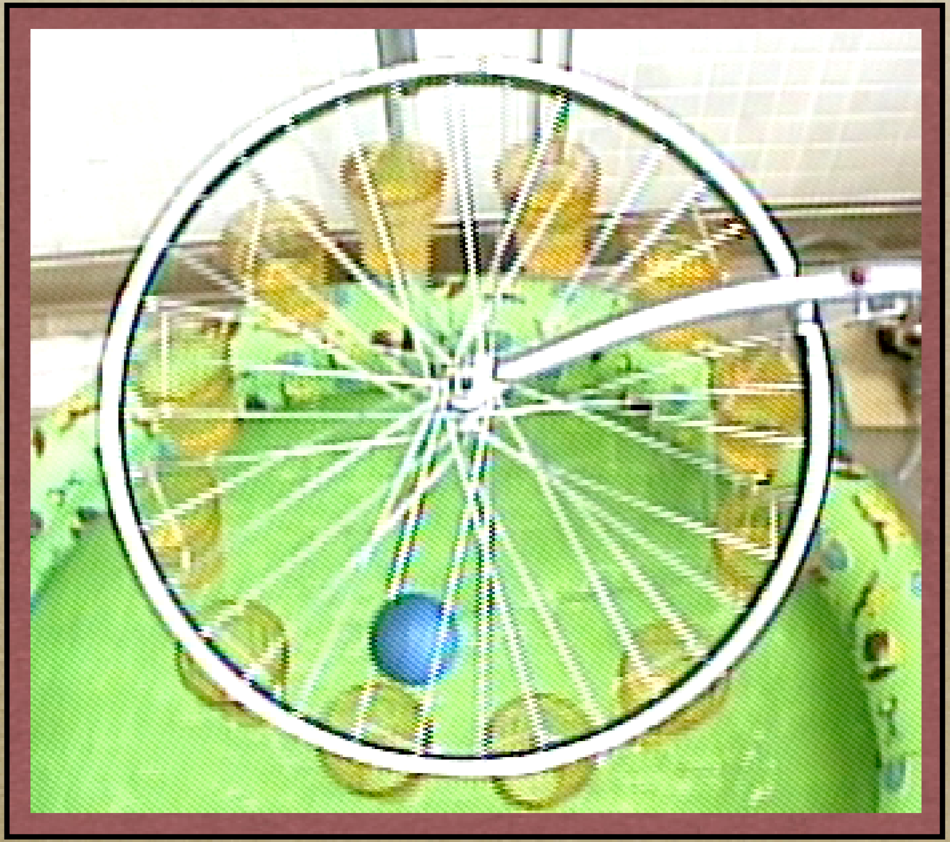
Wheel Diameter	25cm
Cup Diameter	6.6cm
Cup Volume	400mL
Inclination Angle	15 deg

- Tracking the fluorescent ball
- color CCD camera (fish eye lens)
- shutter speed = 1/2000 s
- NI frame grabber + LabView 6.0



Waterwheel in Action

Watch for the change in behavior



Equations of Motion

1) Angle change for each cup

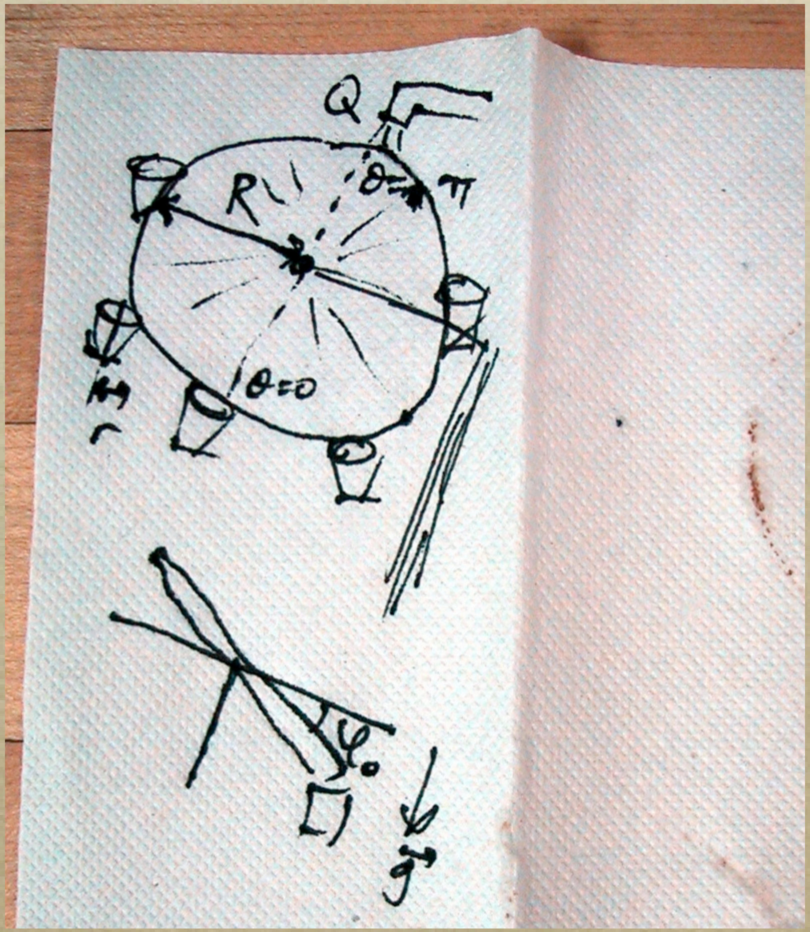
$$\frac{d\theta_i}{dt} = \omega$$

2) Mass change in each cup

$$\frac{dm_i}{dt} = -Leak(m_i) + Fill(\theta_i)$$

3) Torque balance of entire wheel

$$\frac{d}{dt}(I\omega) = gravitational - friction$$



Equations of Motion

1) Angle change for each cup

$$\frac{d\theta_i}{dt} = \omega$$

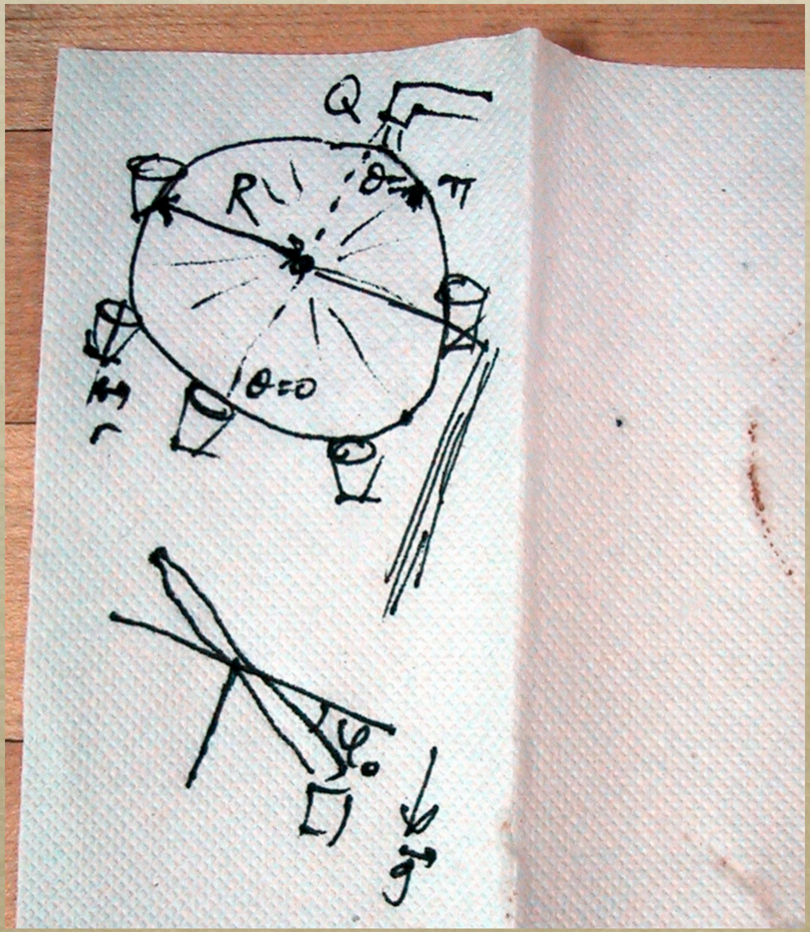
2) Mass change in each cup

$$\frac{dm_i}{dt} = -a\sqrt{\frac{2g\rho m_i}{\pi r^2}} + Q(\sin(\frac{\theta_i}{2}))^n$$

Note: $Q = 0$ for $m > m_{max}$

3) Torque balance of entire wheel

$$\frac{d}{dt}(I\omega) = gravitational - friction$$



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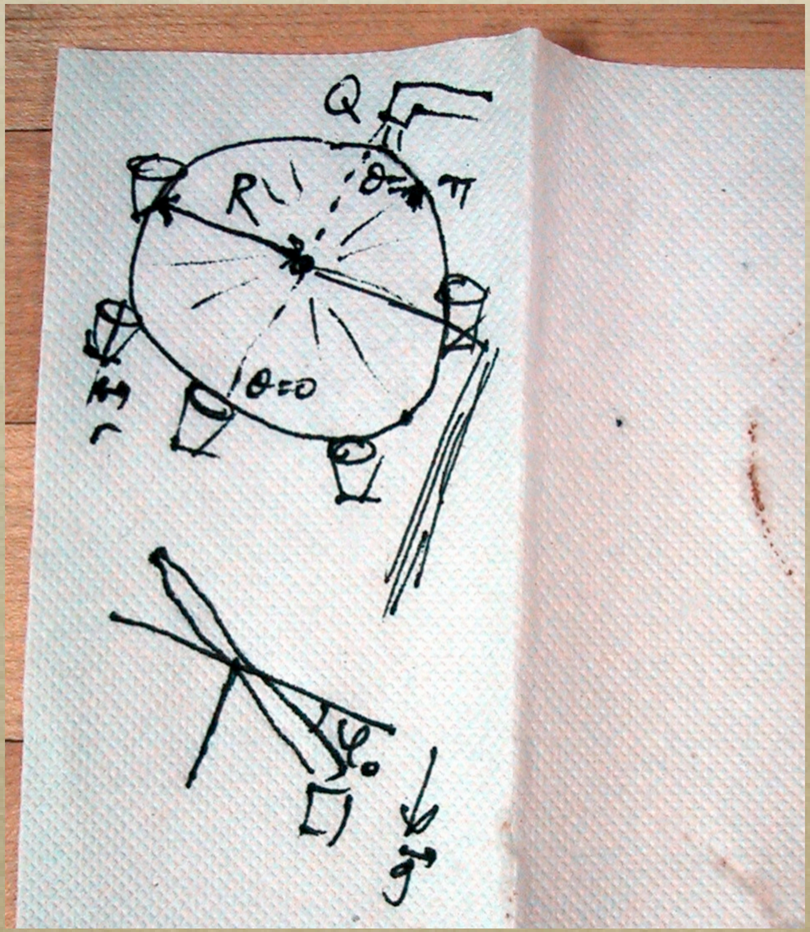
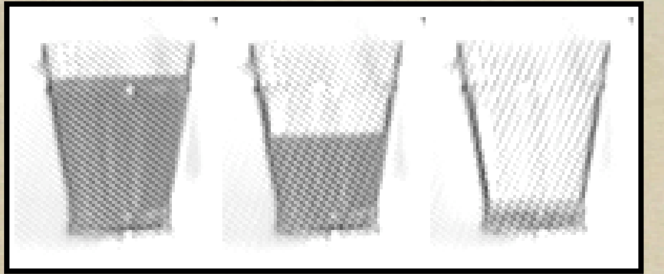
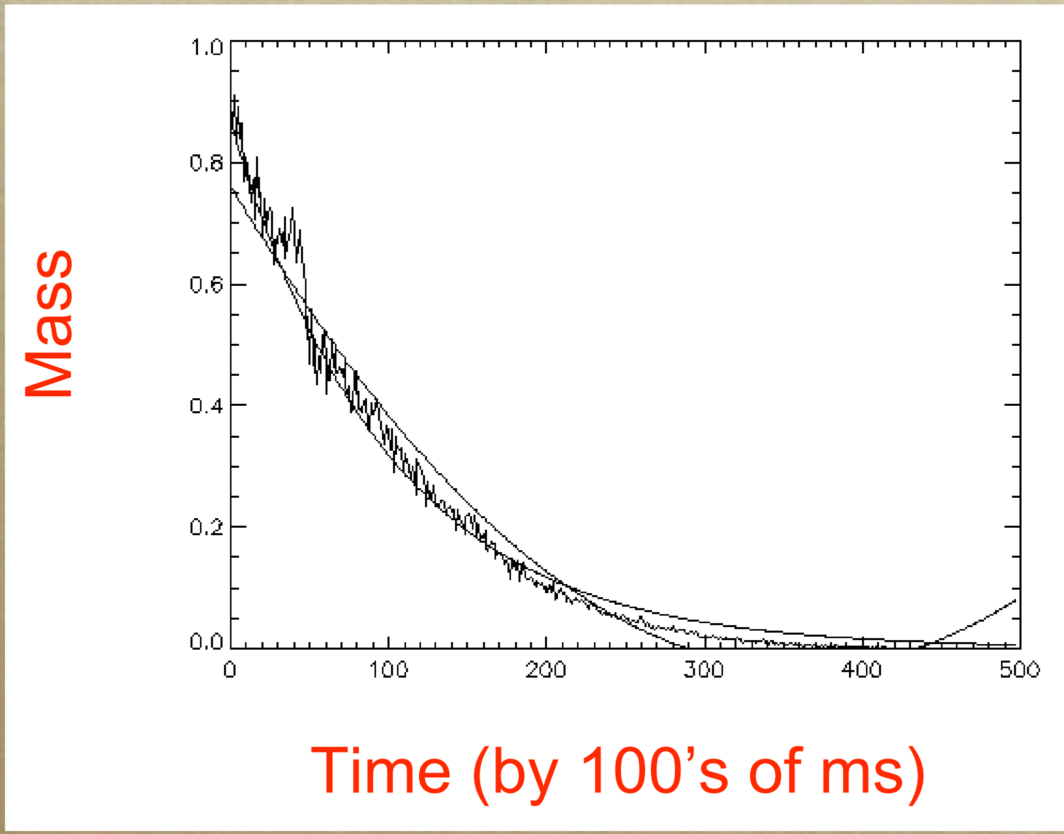
3) Torque balance of entire wheel

$$\frac{d\omega}{dt}\sum_{i=1}^{N} m_i R^2 + \omega \sum_{i=1}^{N} \frac{dm_i}{dt} R^2 = Rg\sin(\varphi_0)\sum_{i=1}^{N} m_i \sin(\theta_i) - \alpha\omega$$



Leak Rate





Our assumption –

Potential energy per unit volume at top of liquid is equal to kinetic energy per unit volume of leaking water.

$$\rho g h = \frac{1}{2}\rho v^2$$

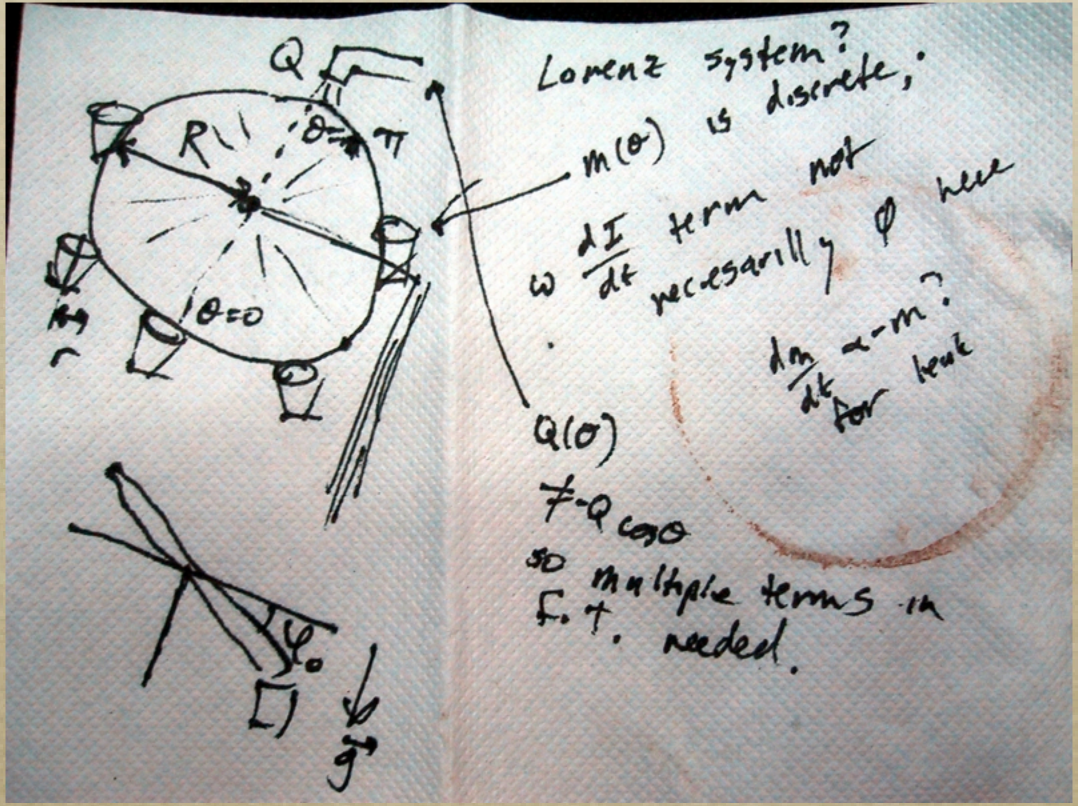
$$h = \frac{m}{\pi r^2 \rho}$$

So…

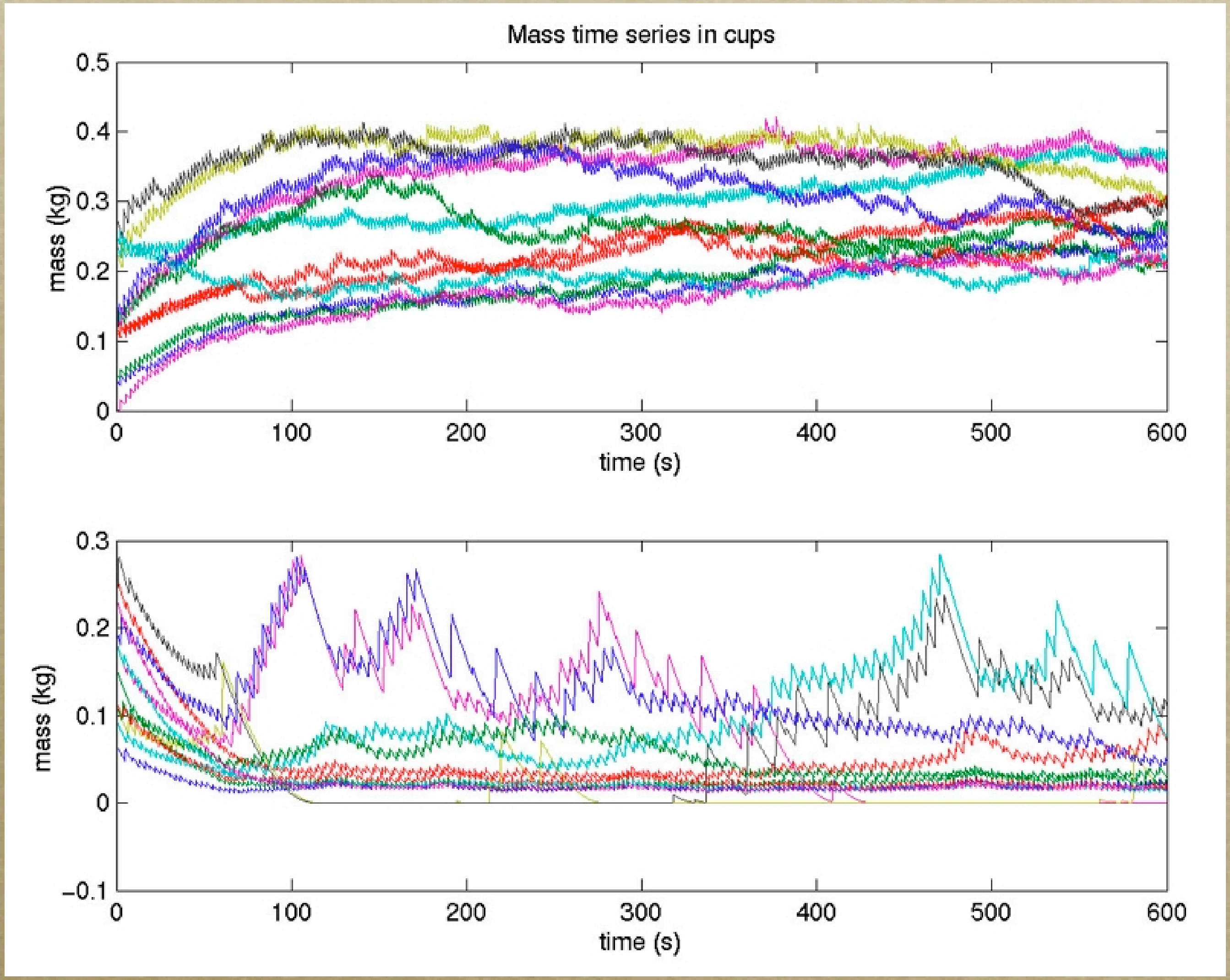
$$\frac{dm}{dt} = -\rho a v = -a\sqrt{\frac{2 g \rho m}{\pi r^2}}$$

Limitations of Strogatz Model

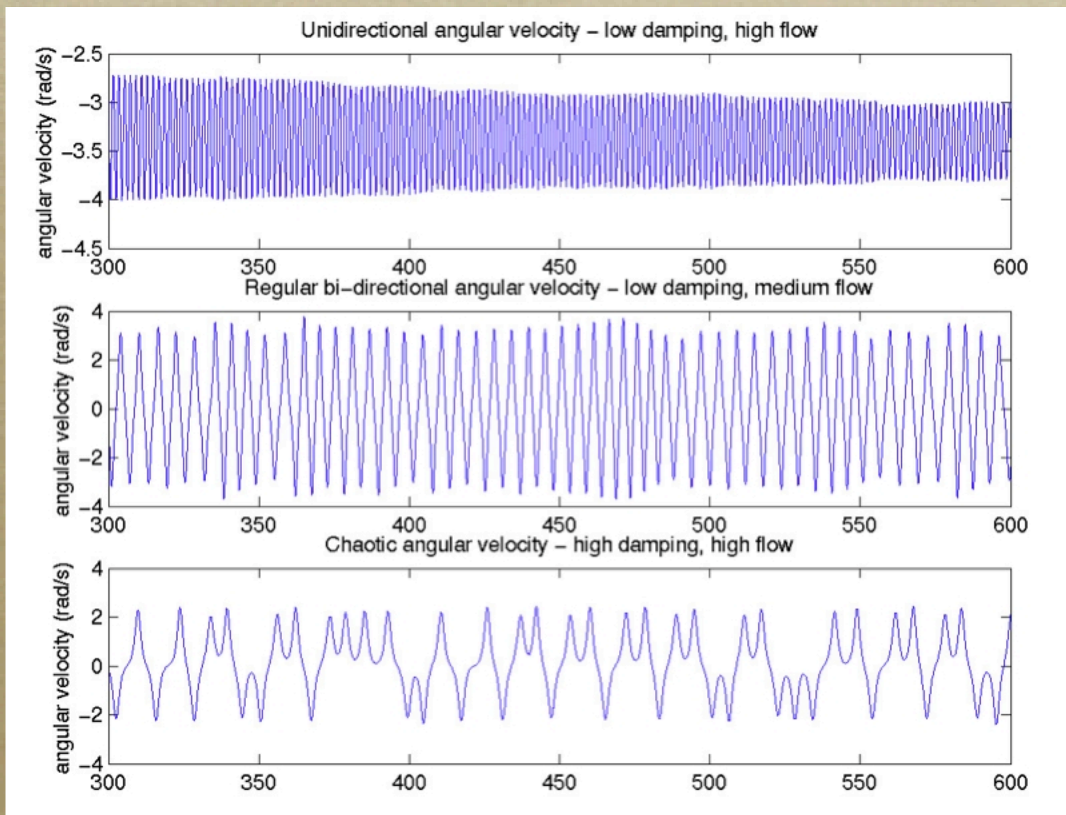
- *Lorenz system*
  - *Discrete vs. continuous distribution of mass*
  - *Take lowest order term in Fourier expansion, then change variables*
- *Completeness of model relative to experiment*



Simulated Mass Regimes

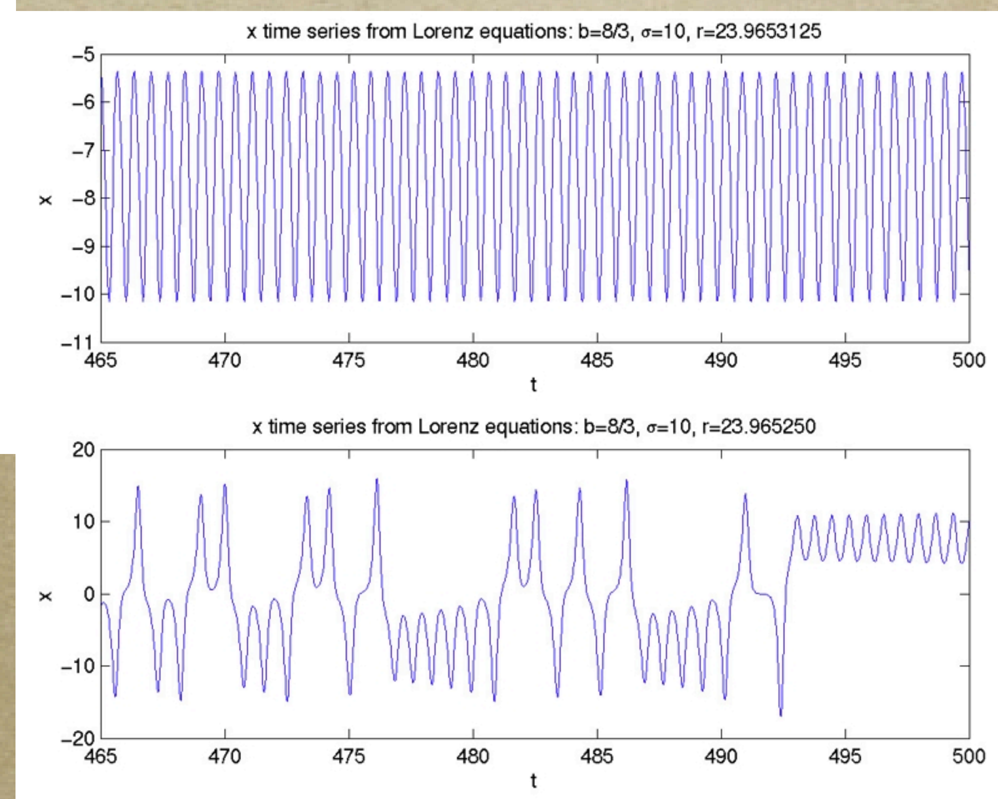


Omega Regimes

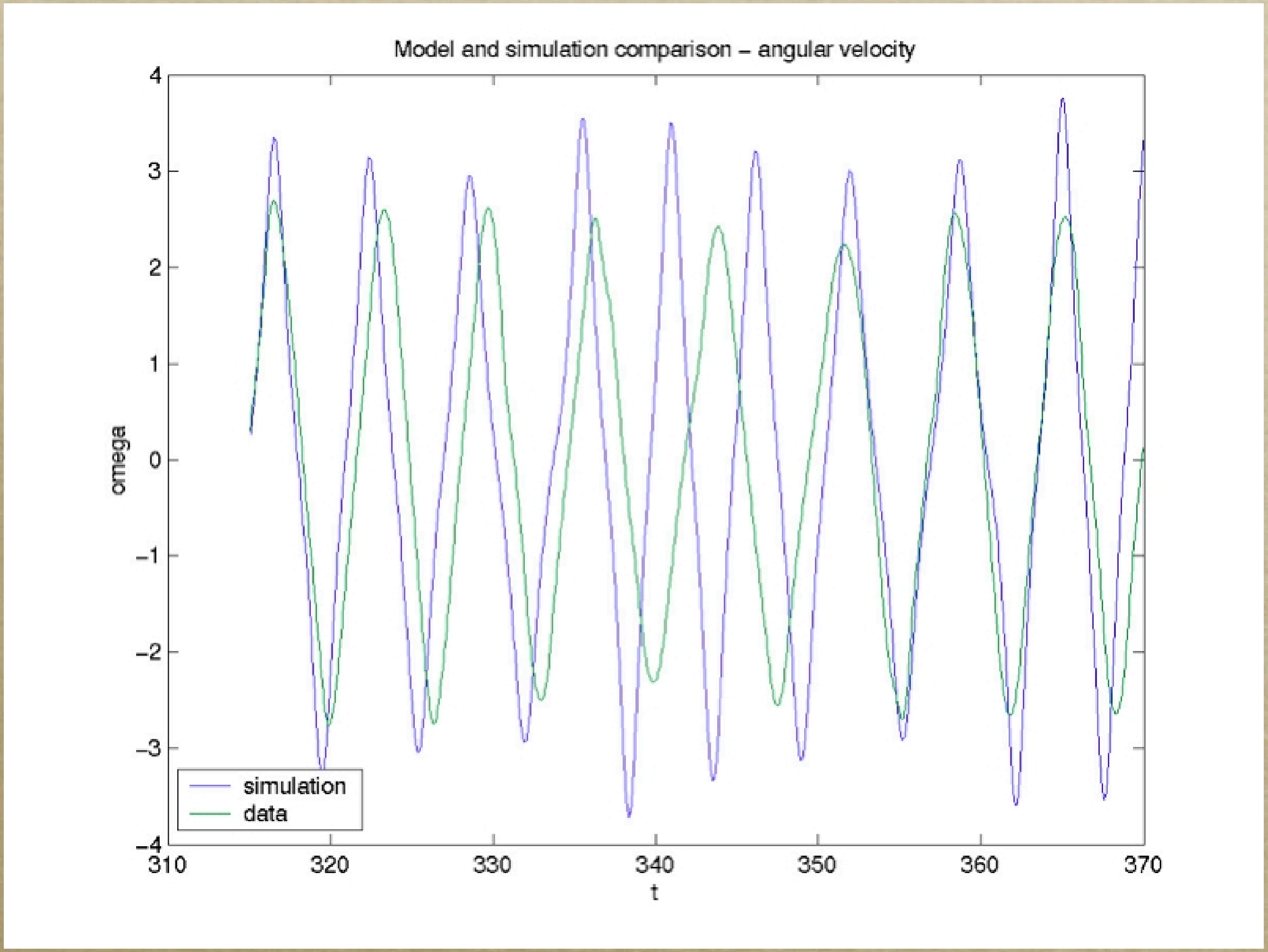


Waterwheel Equations

Lorenz Equations

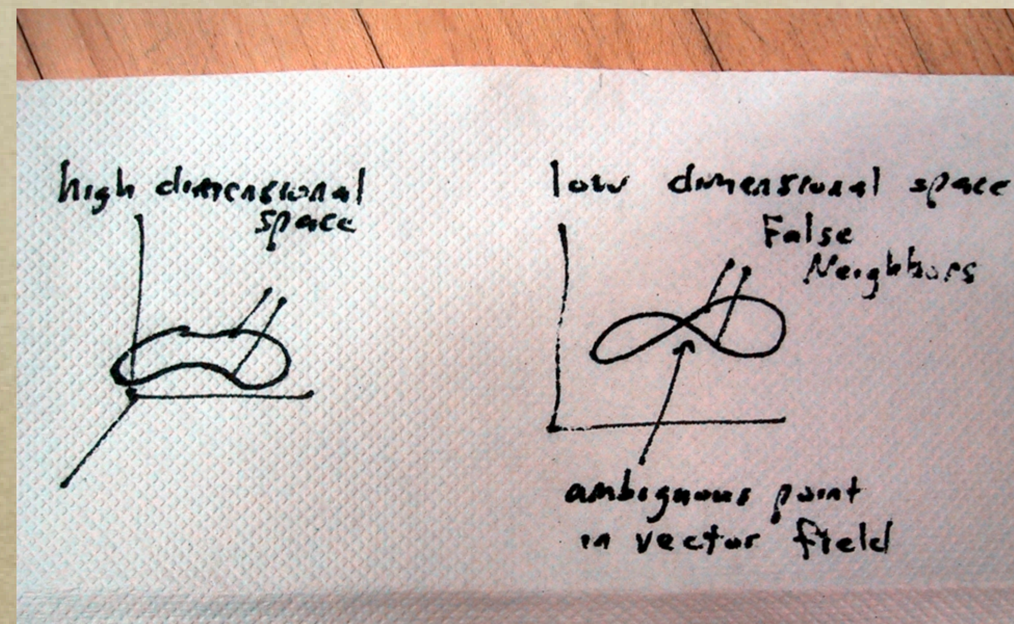


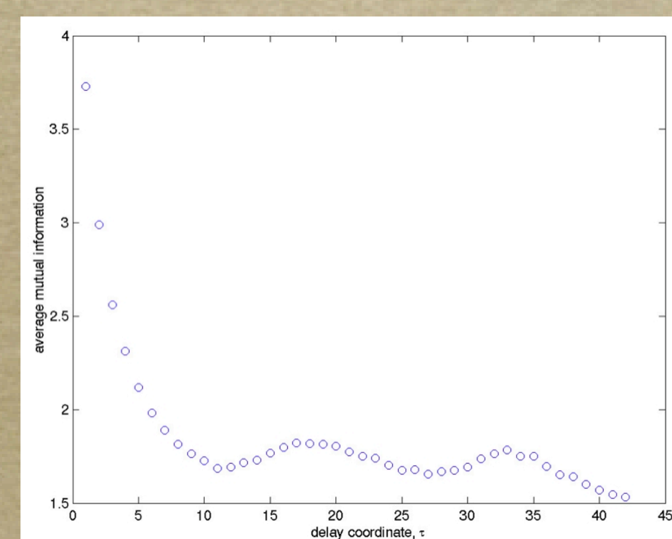
Model Agreement



Phase Space Reconstruction

- *reconstruction preserves topological features*
- *delay coordinate (tau) embedding*
    - *average mutual entropy*
- *global false nearest neighbors,* d_E
- d_E *not associated with dimensionality of original system*

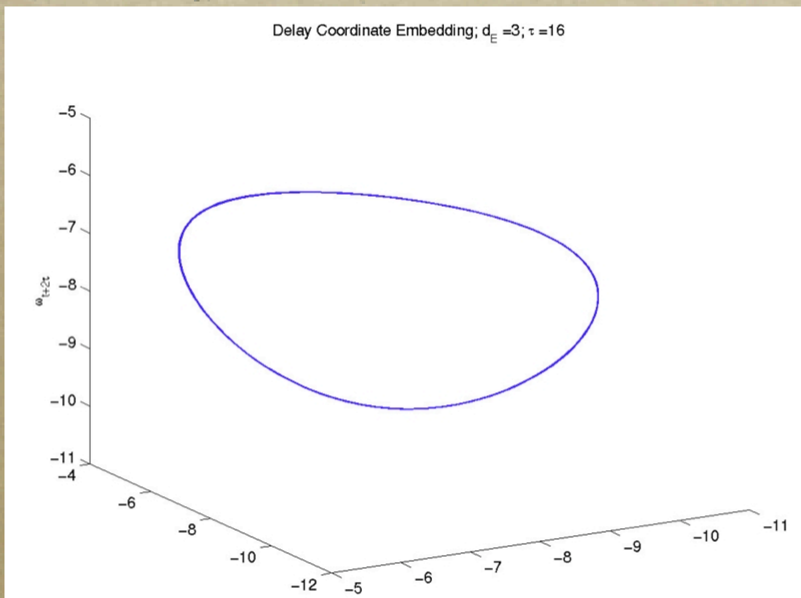


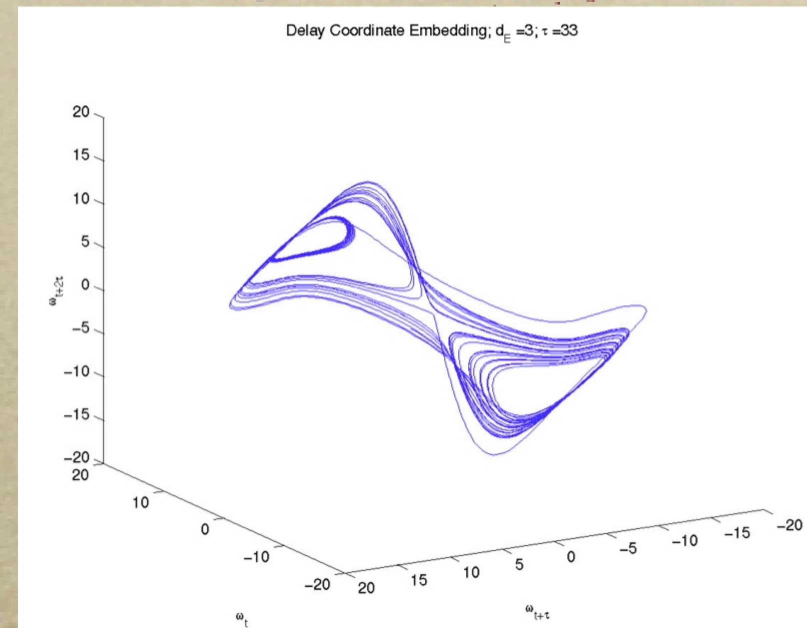


$$y(k) = [s(k),\ s(k+T),\ \ldots,\ s(k+(d-1)T\ ]$$

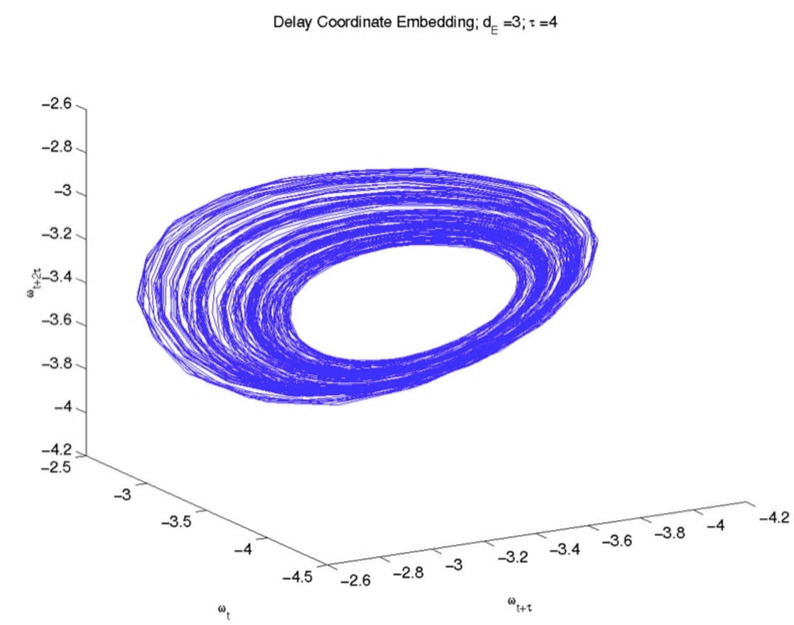
$$y^{NN}(k) = [s^{NN}(k),\ s^{NN}(k+T),\ \ldots,\ s^{NN}(k+(d-1)T]$$

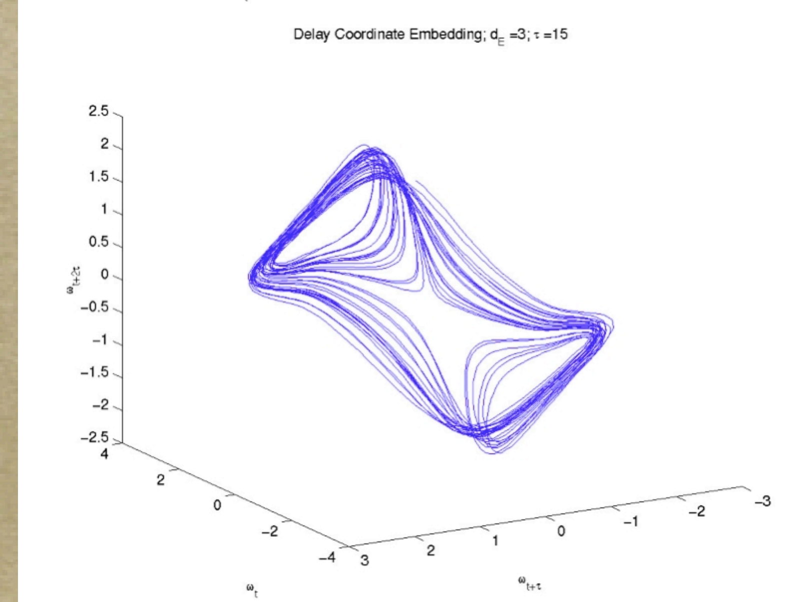
Lorenz and Model Attractors





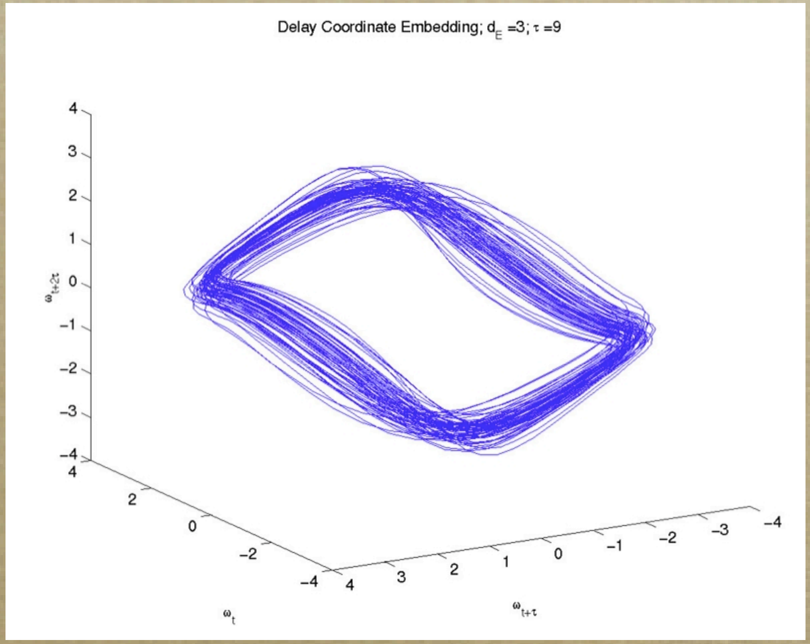
Lorenz



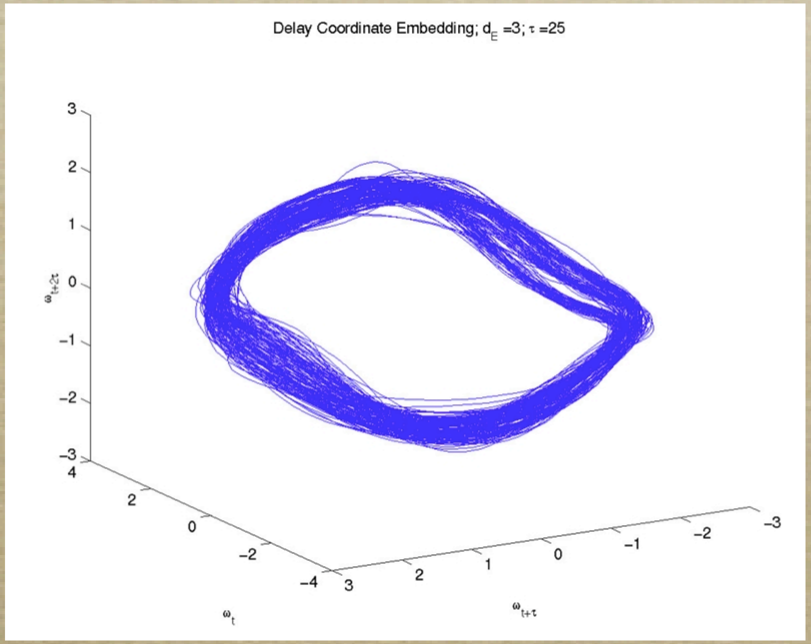


Model

Model/Reality Agreement



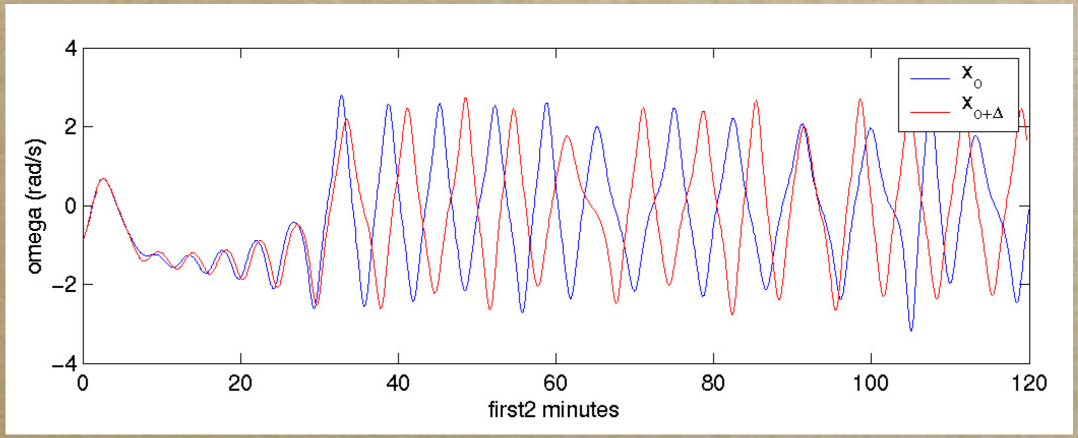
Model



Data

Sensitivity to Initial Conditions

- *simulating x_0 and x_0+delta*
- *local Lyapunov exponent - nearby points separate exponentially in time*



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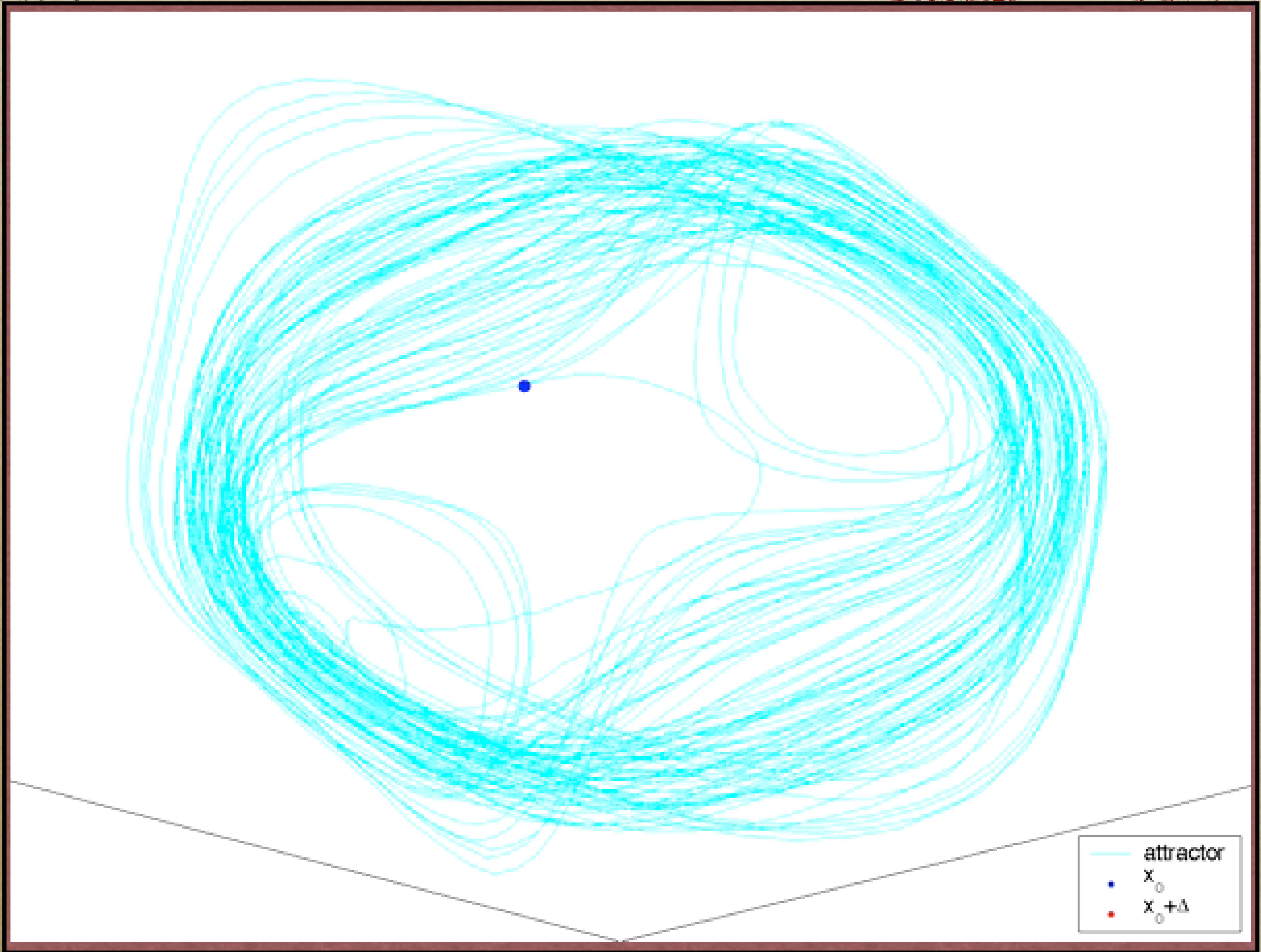

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