# The Logistic Flow

(Continuous)

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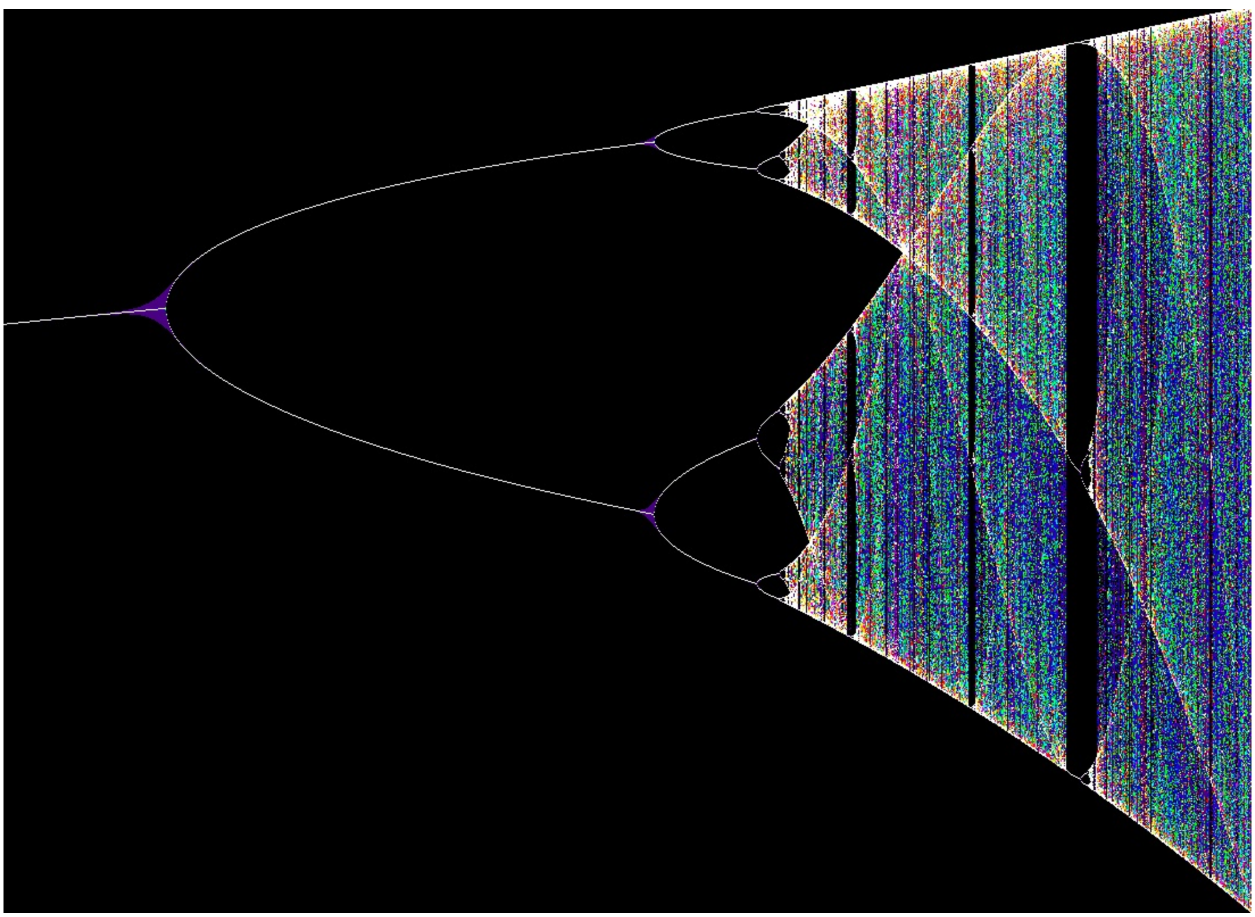
# Logistic flow . . . ←

We all know that the discrete logistic map

$$P_{n+1} \;=\; rP_n(1 - P_n)$$

exhibits interesting behavior of various sorts for various values of the parameter $r$, including chaos, etc.



What kind of behavior can we expect for the continuous version of a logistic flow:

$$\frac{dP}{dt} = rP(1-P) \quad ?$$

Note that this is a non-linear ODE, but fortunately we can actually integrate . . .

$$\begin{aligned} \frac{dP}{dt} &= rP(1-P) \\ \frac{dP}{P(1-P)} &= rdt \end{aligned}$$

Thus:

$$\begin{aligned} \int \frac{dP}{P(1-P)} &= \int rdt \\ \int \frac{dP}{P(1-P)} &= rt + c_1 \end{aligned}$$

By partial fractions, we have:

$$\begin{aligned}
\int \frac{dP}{P} + \int \frac{dP}{(1-P)} &= rt + c_1 \\
\log(P) - \log(1-P) &= rt + c_1 \\
\log(\frac{P}{1-P}) &= rt + c_1 \\
\frac{P}{1-P} &= e^{rt+c_1} \\
\frac{P}{1-P} &= c_2 e^{rt} \\
P &= (1-P) c_2 e^{rt}
\end{aligned}$$

And thus:

$$\begin{aligned} P &= (1-P)c_2e^{rt} \\ P &= c_2e^{rt} - Pc_2e^{rt} \\ P + Pc_2e^{rt} &= c_2e^{rt} \\ P(1+c_2e^{rt}) &= c_2e^{rt} \end{aligned}$$

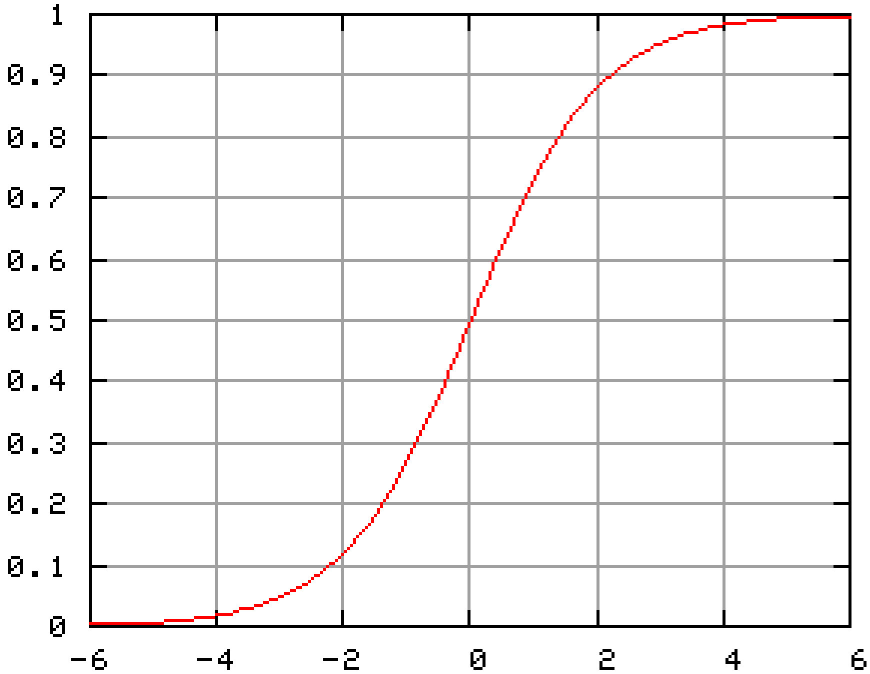
giving us:

$$P = \frac{c_2e^{rt}}{1+c_2e^{rt}}$$

and, dividing top and bottom by $c_2e^{rt}$, and simplifying, we have:

$$P = \frac{1}{1+ce^{-rt}}$$

This function just gives us the classic logistic/sigmoid curve:




and changes in $c$ and $r$ make minor changes in the behavior near 0 . . .

The difference between the behavior of the discrete and continuous logistic functions can give us some idea of the significance of working in the discrete regime . . .

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