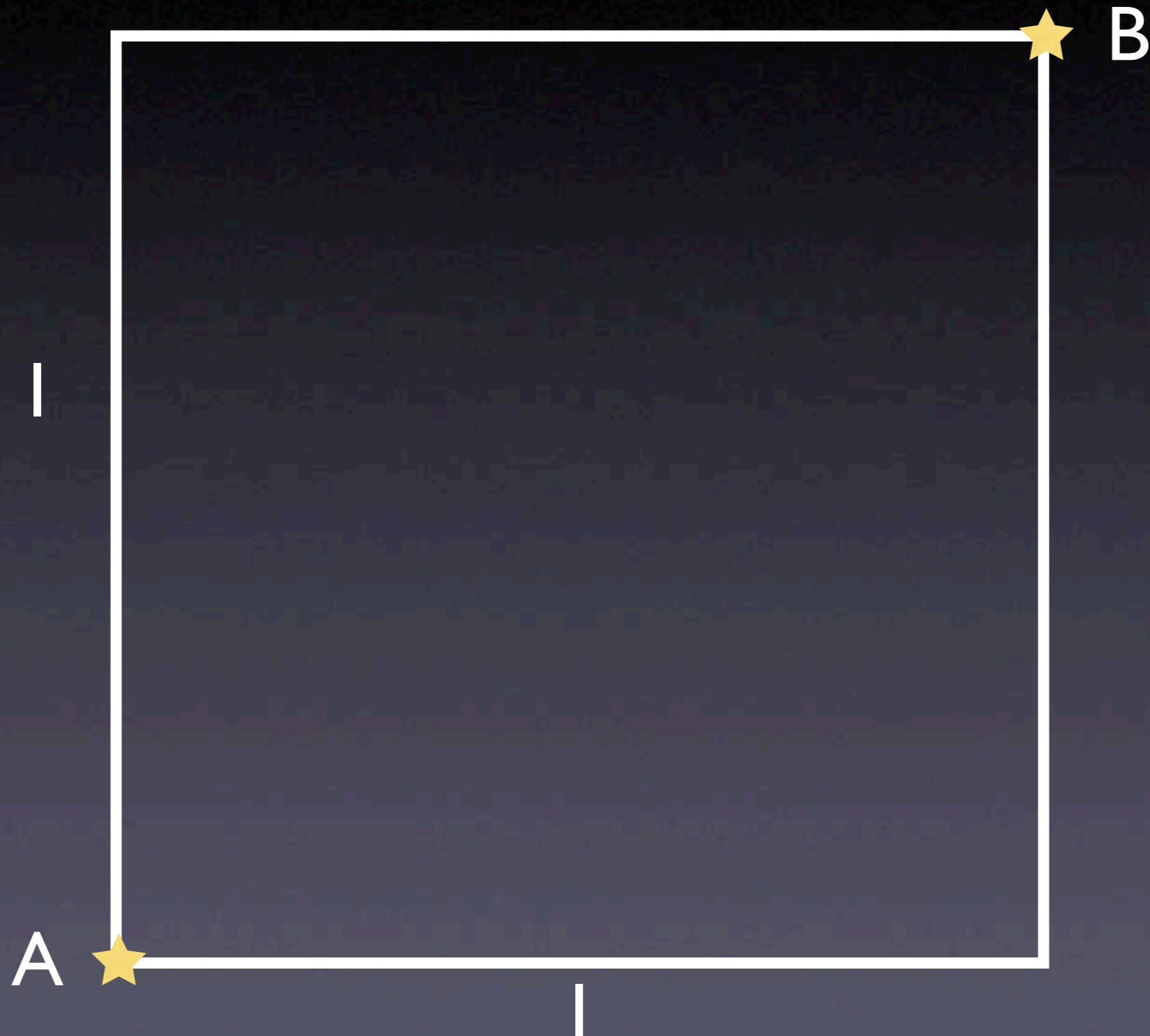


Discrete vs. Continuous
(taxicab geometry)

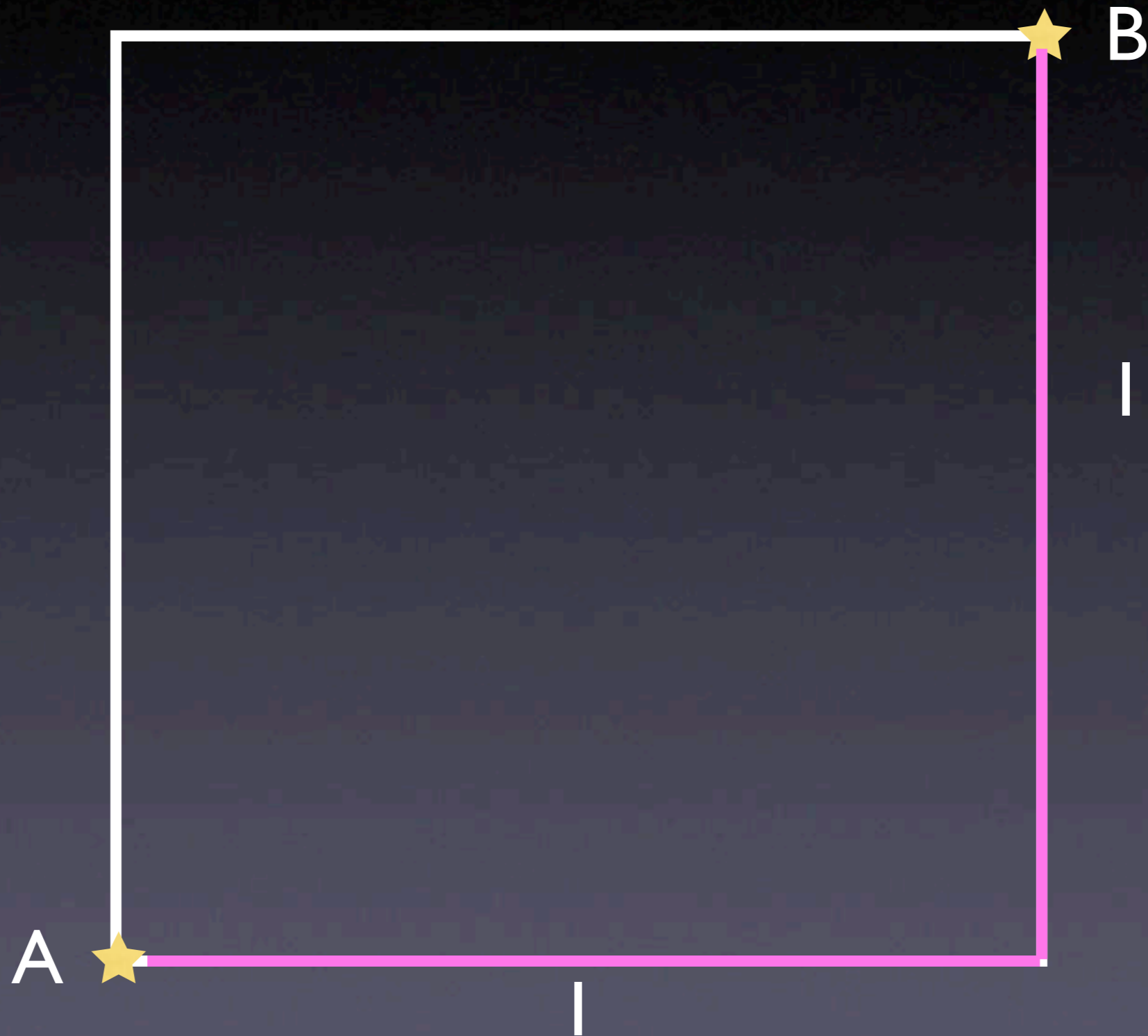
- There is a strong tradition within academia (and science in particular) to assume that most of the time, it is OK to use a continuous model of a system, even if the system at issue is inherently discrete. The typical “justification” is that “in the limit, discrete becomes continuous ...”

- What follows here is sort of a joke, but contains ideas worth thinking about . . .
- Let's consider "distance" -- but in a special case. The "geometry" here is often called "taxicab geometry," using "taxicab distance."

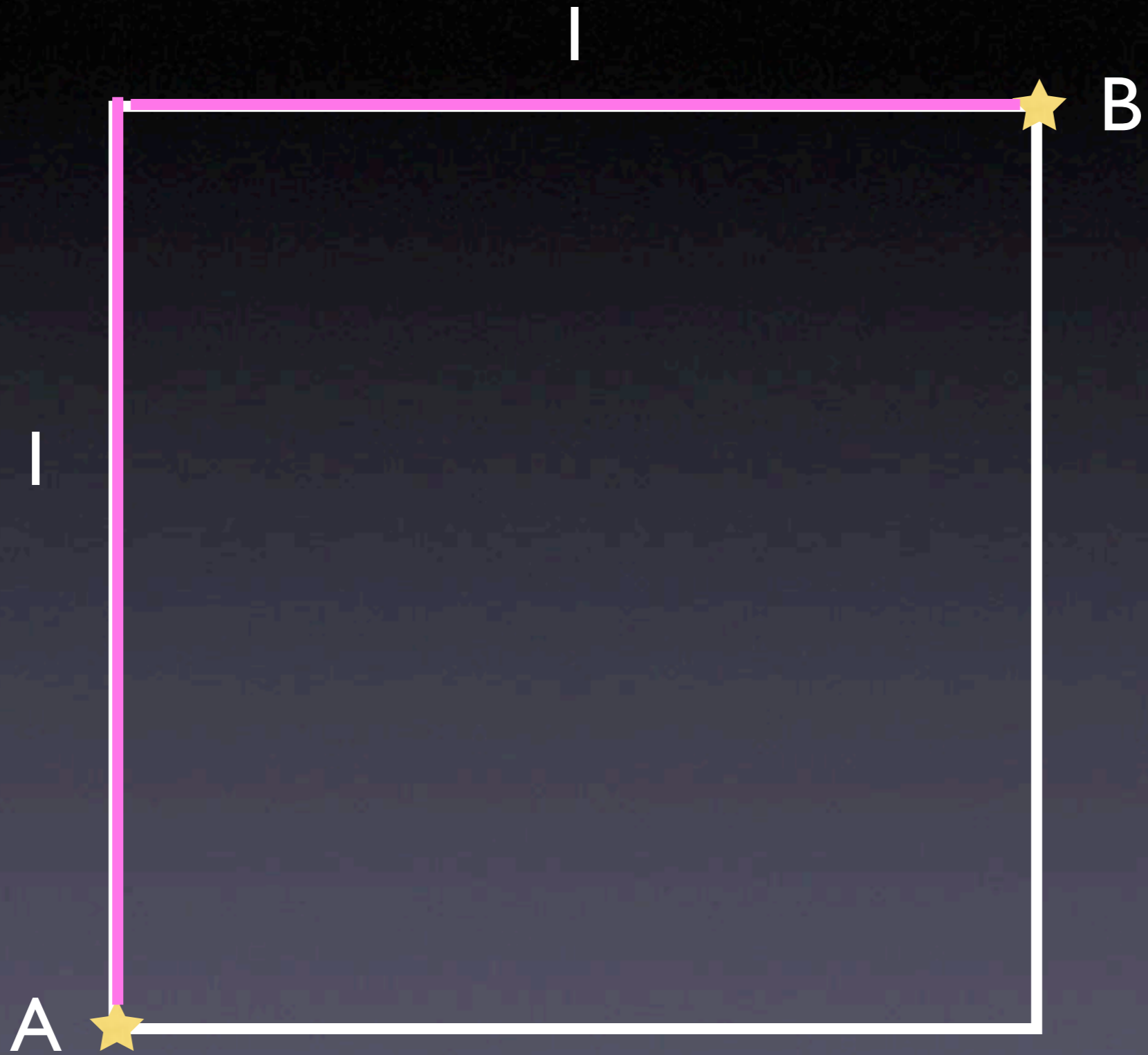
- Consider “distance” in a city, where we must follow streets. We want to go from A to B:



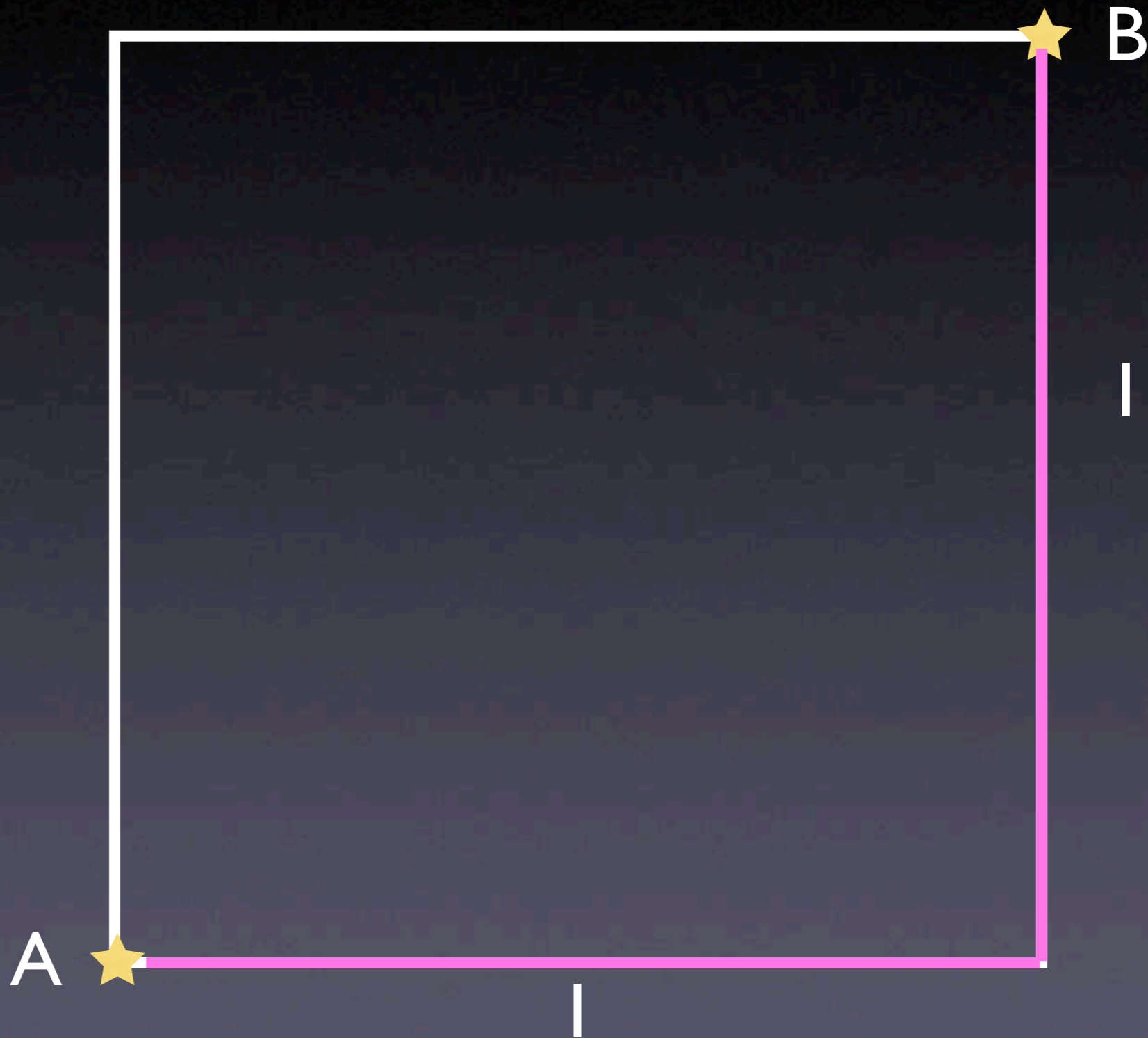
- The sides of the square are length l , and we can either go this way:



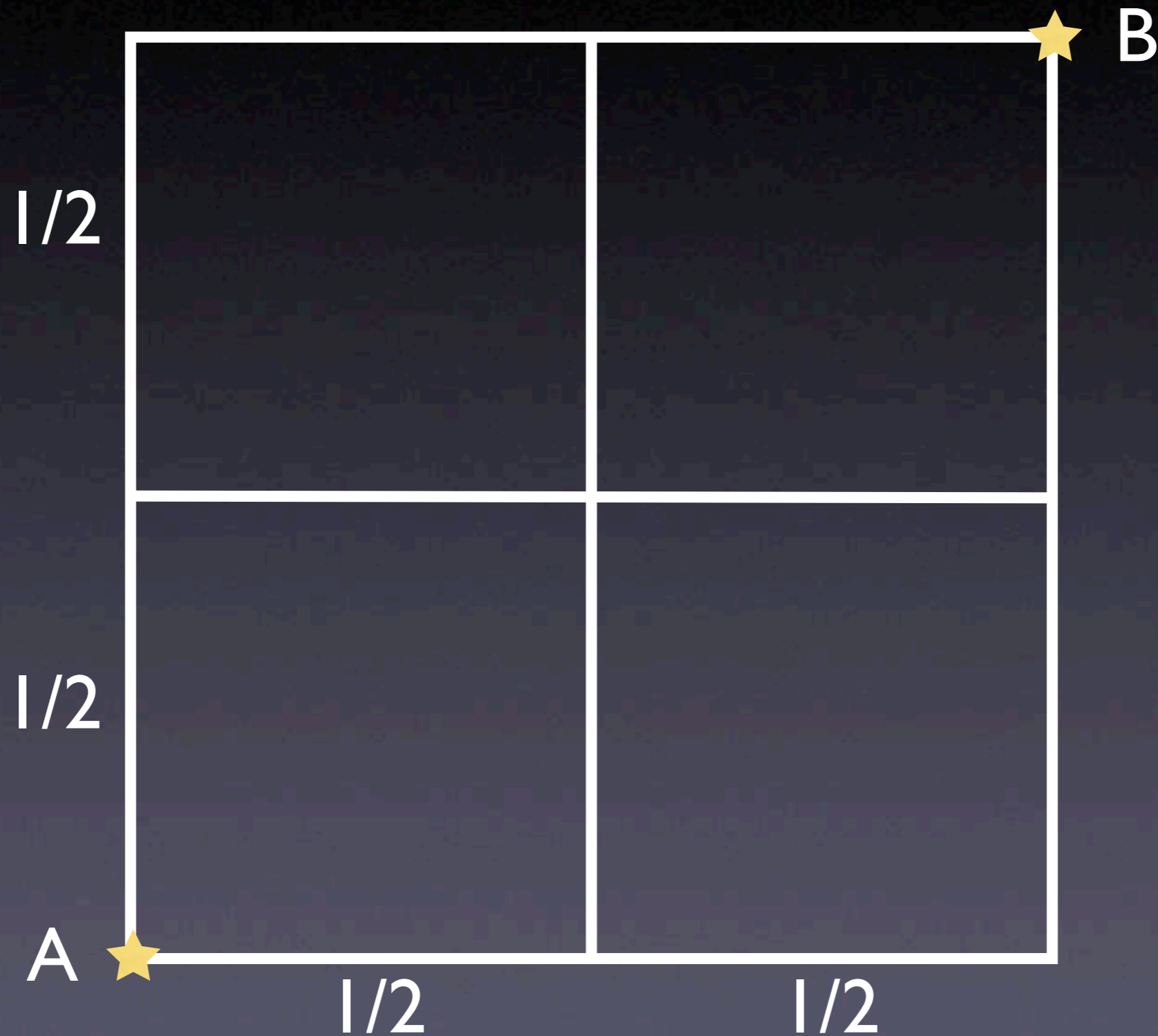
- Or this way:



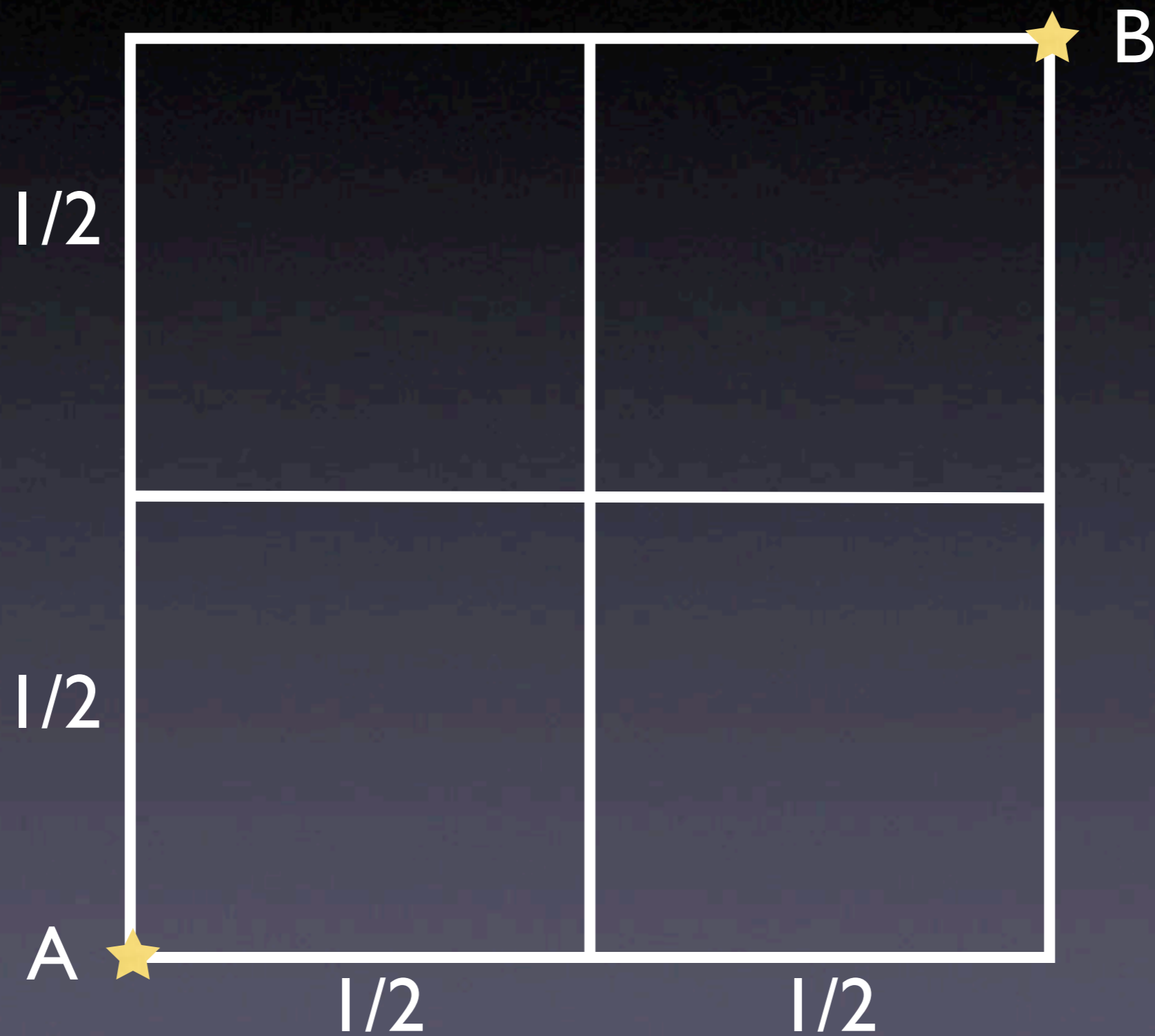
- But either way, the “taxicab distance” from A to B is 2:



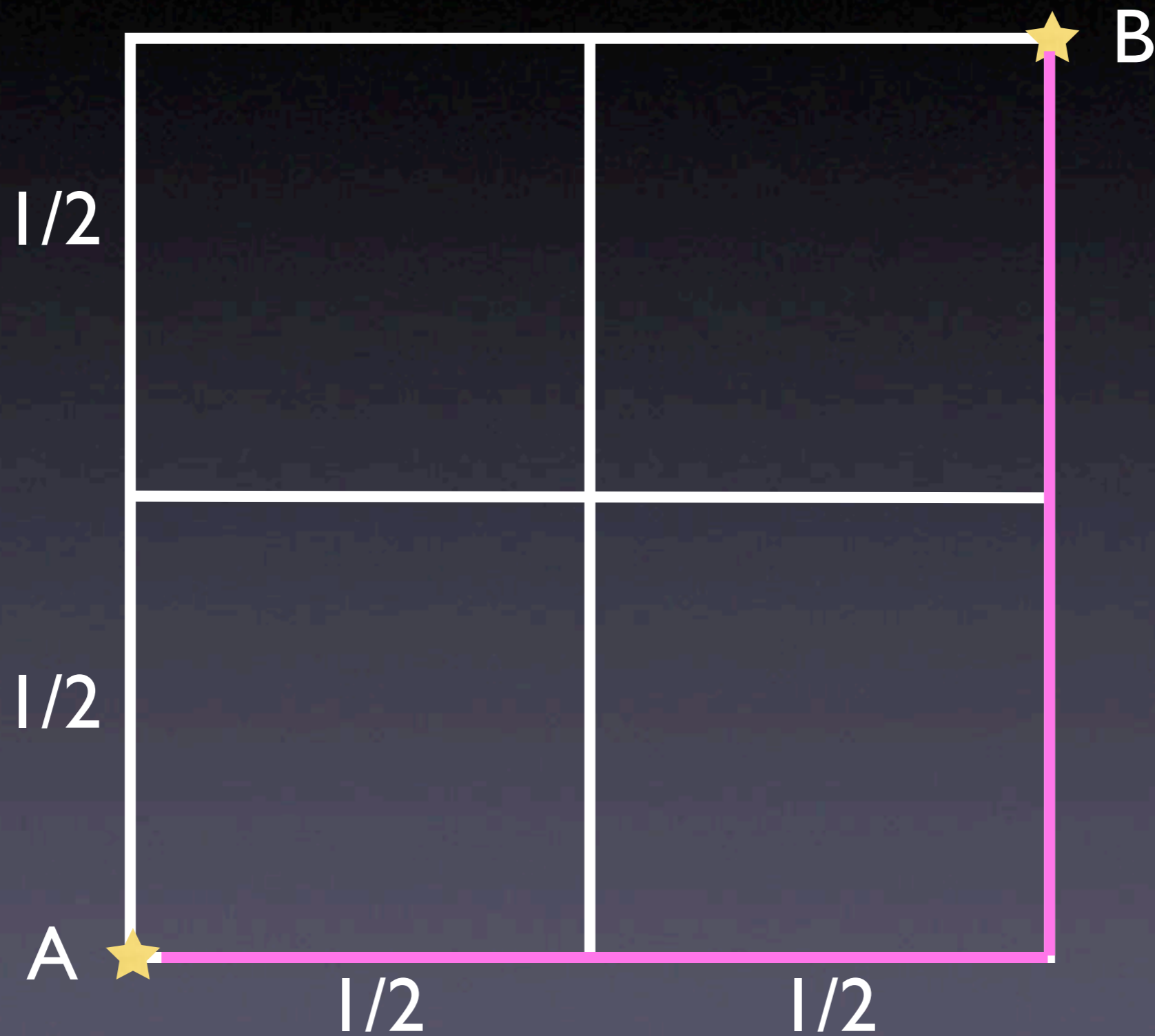
- Now suppose there is “redevelopment,” and the city block is subdivided:



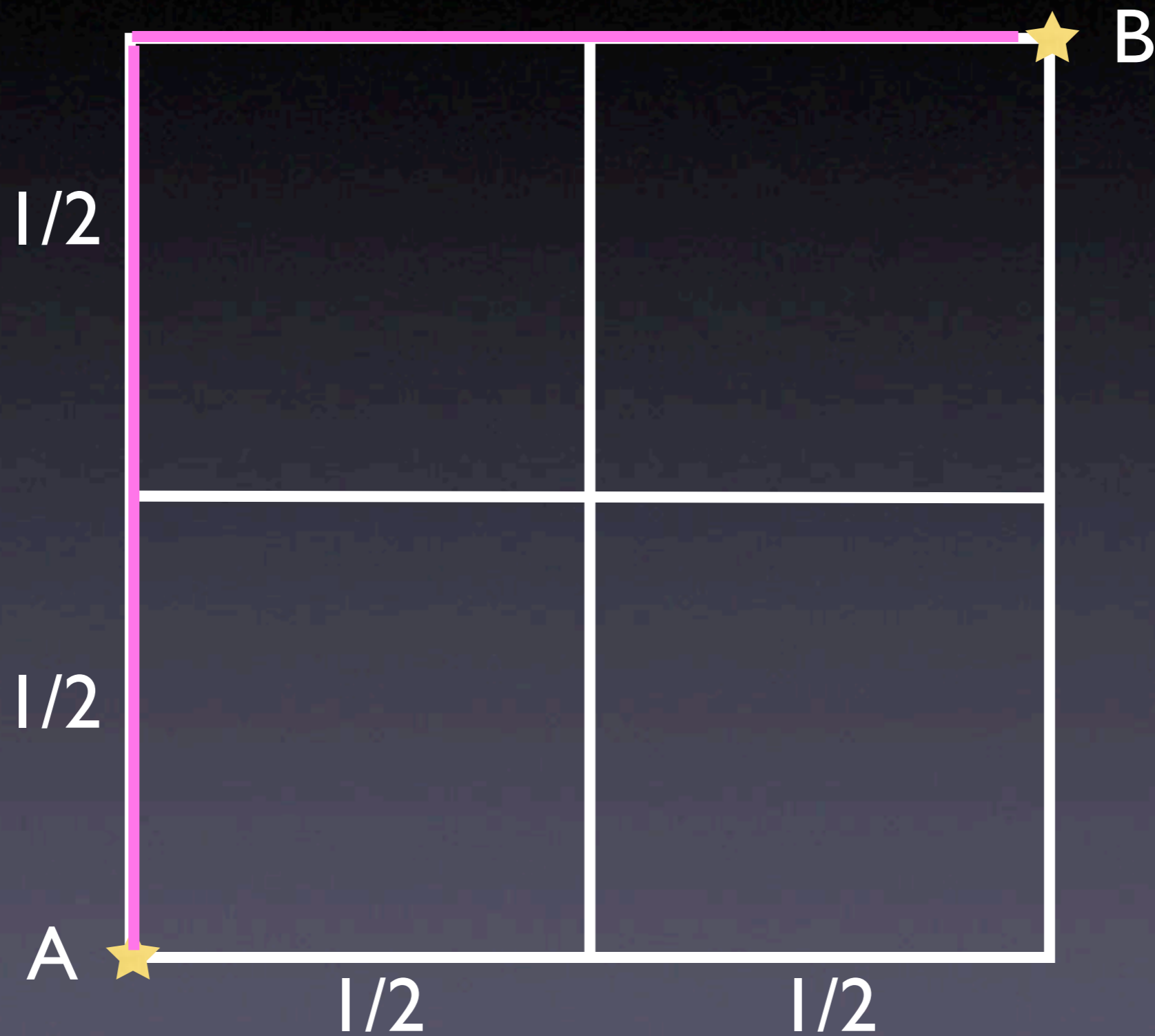
- We can go from *A* to *B* in various ways:



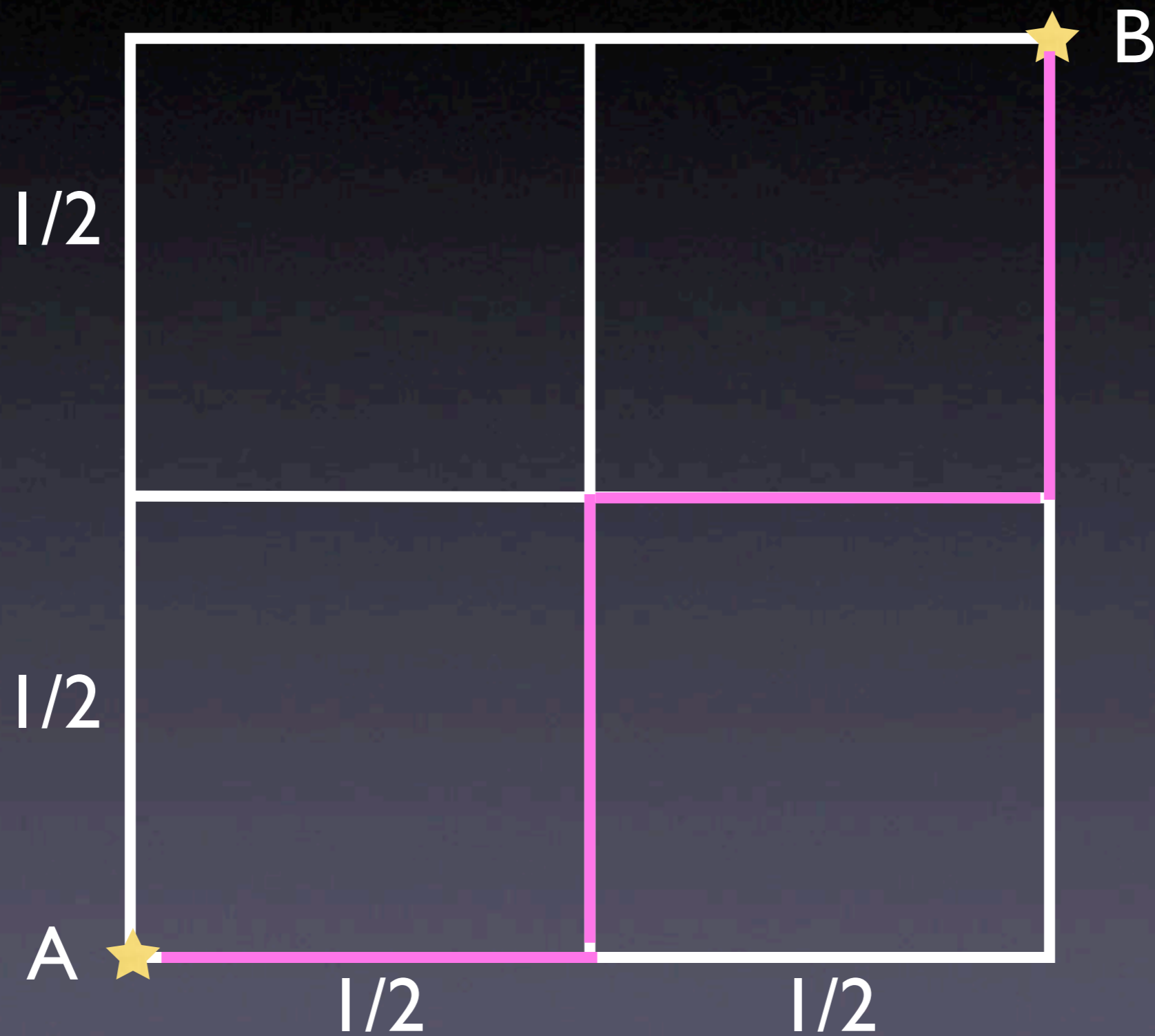
- We can go from A to B in various ways:



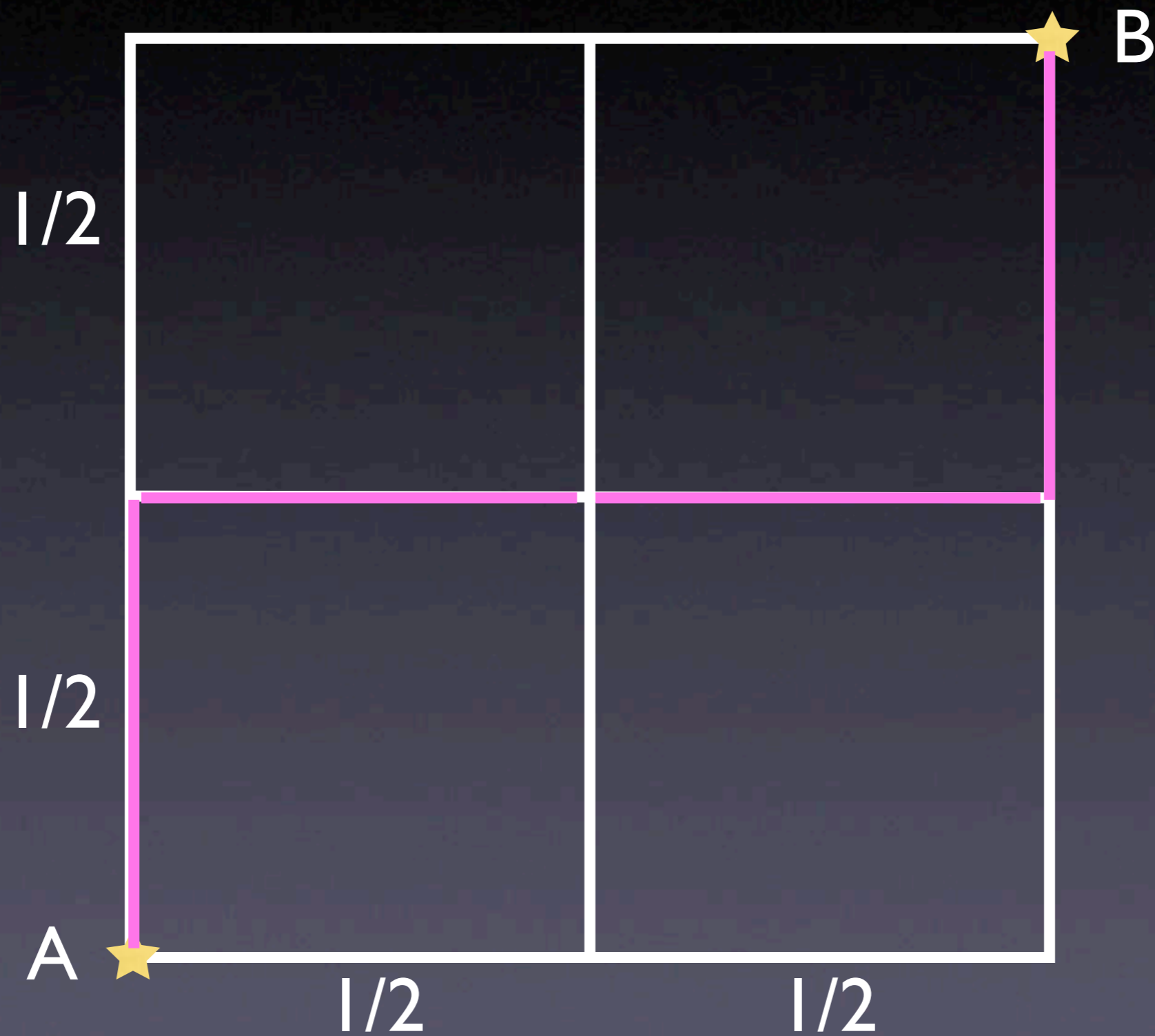
- We can go from A to B in various ways:



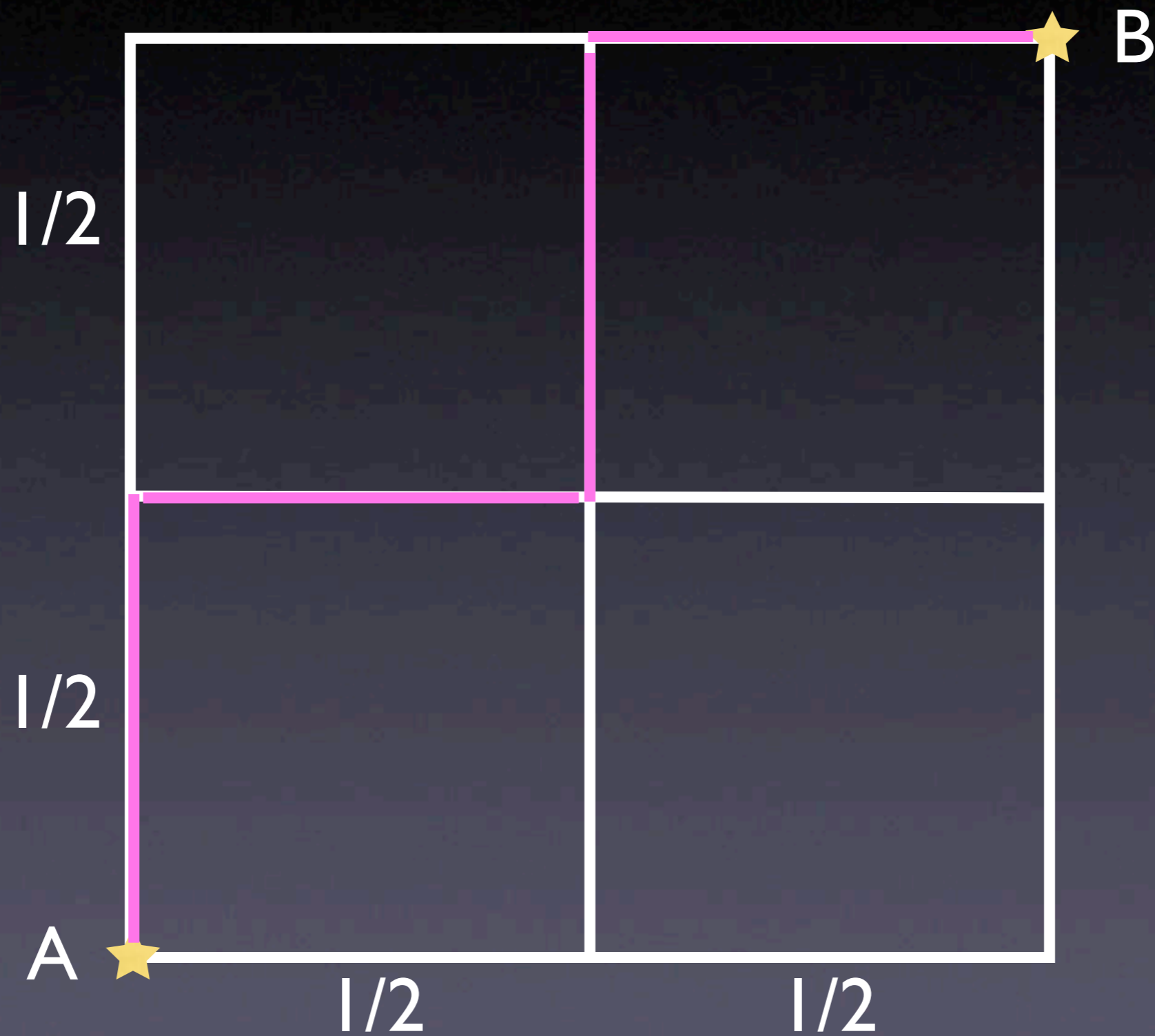
- We can go from A to B in various ways:



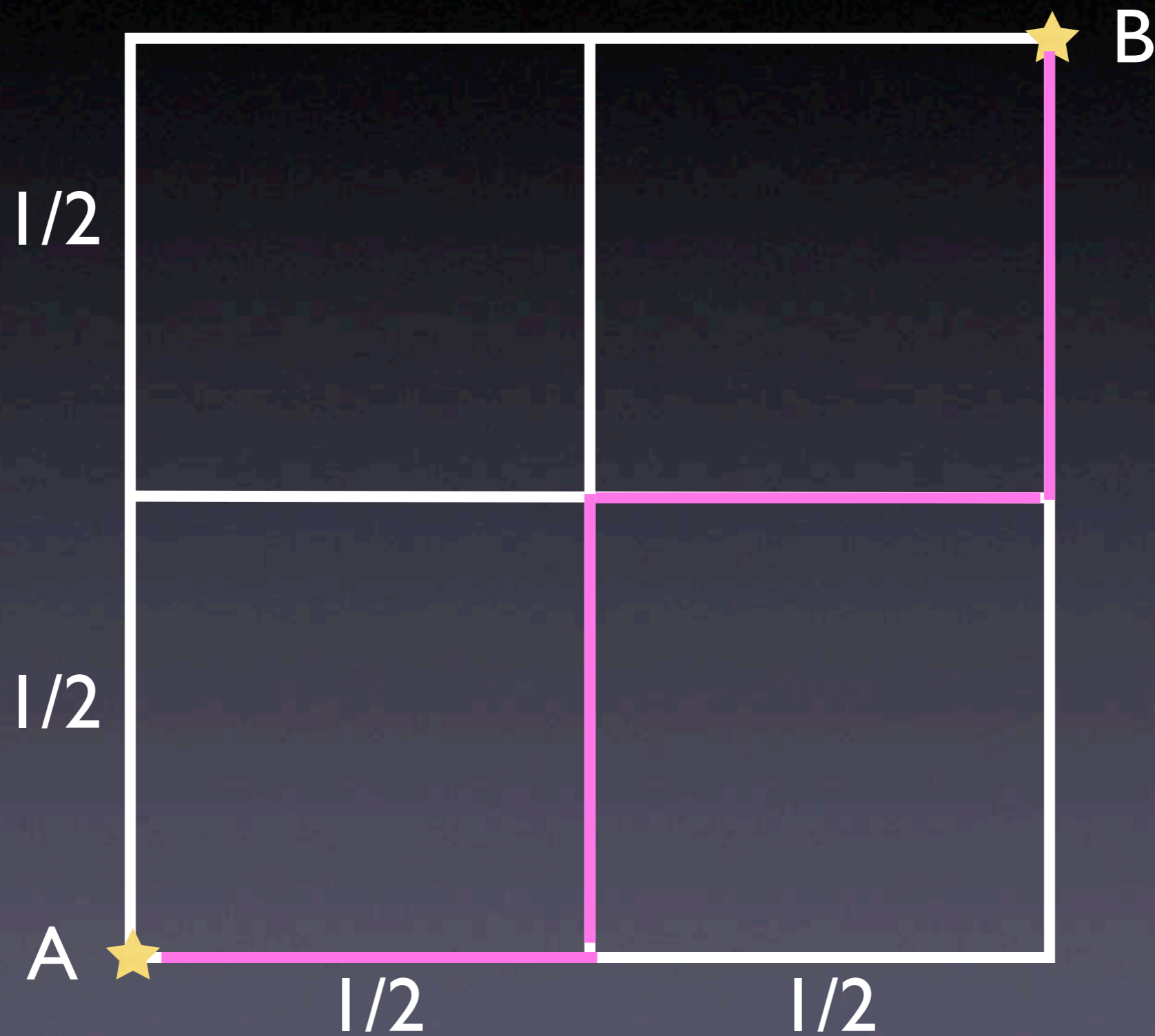
- We can go from A to B in various ways:



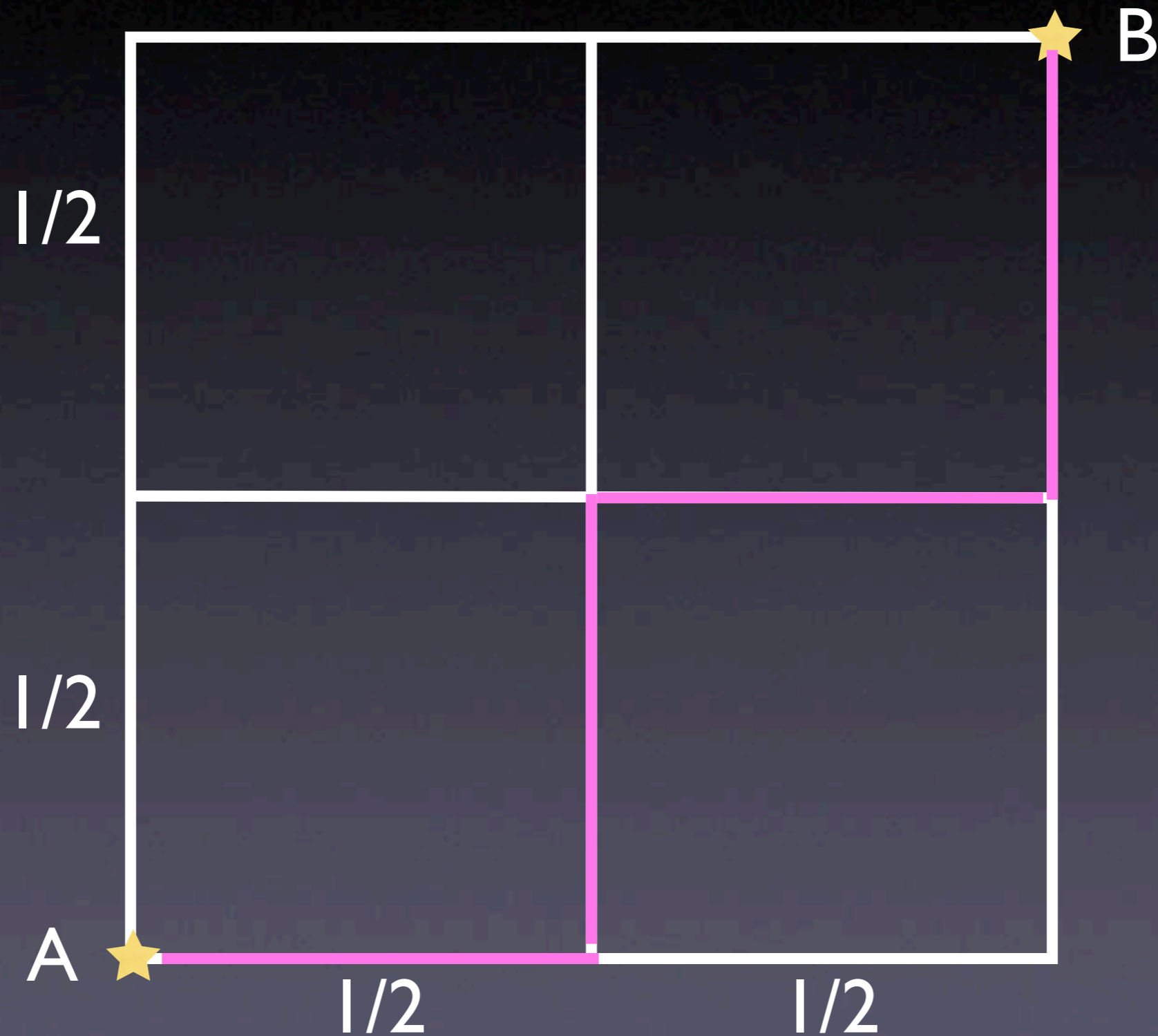
- We can go from A to B in various ways:



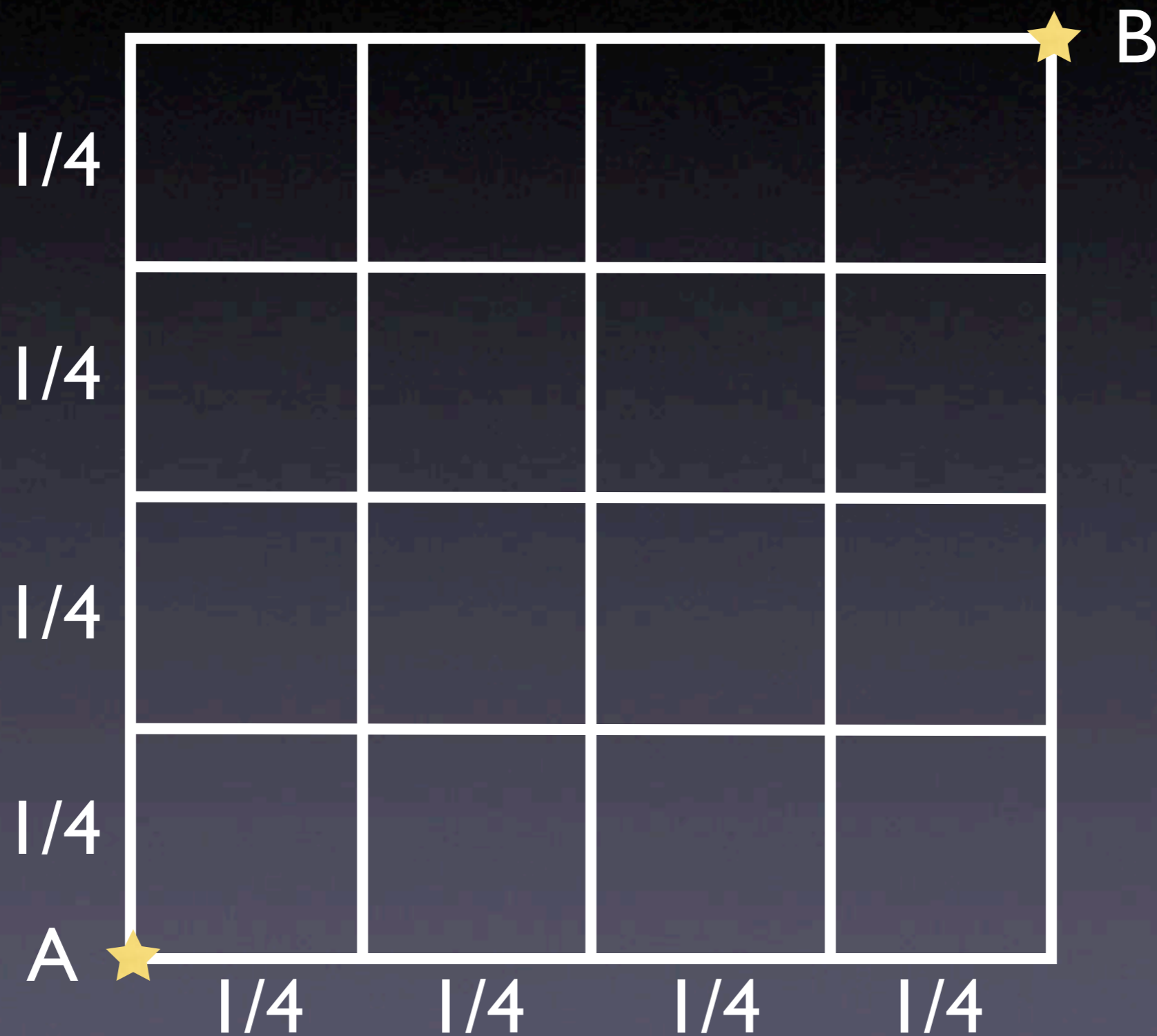
- We can go from A to B in various ways:



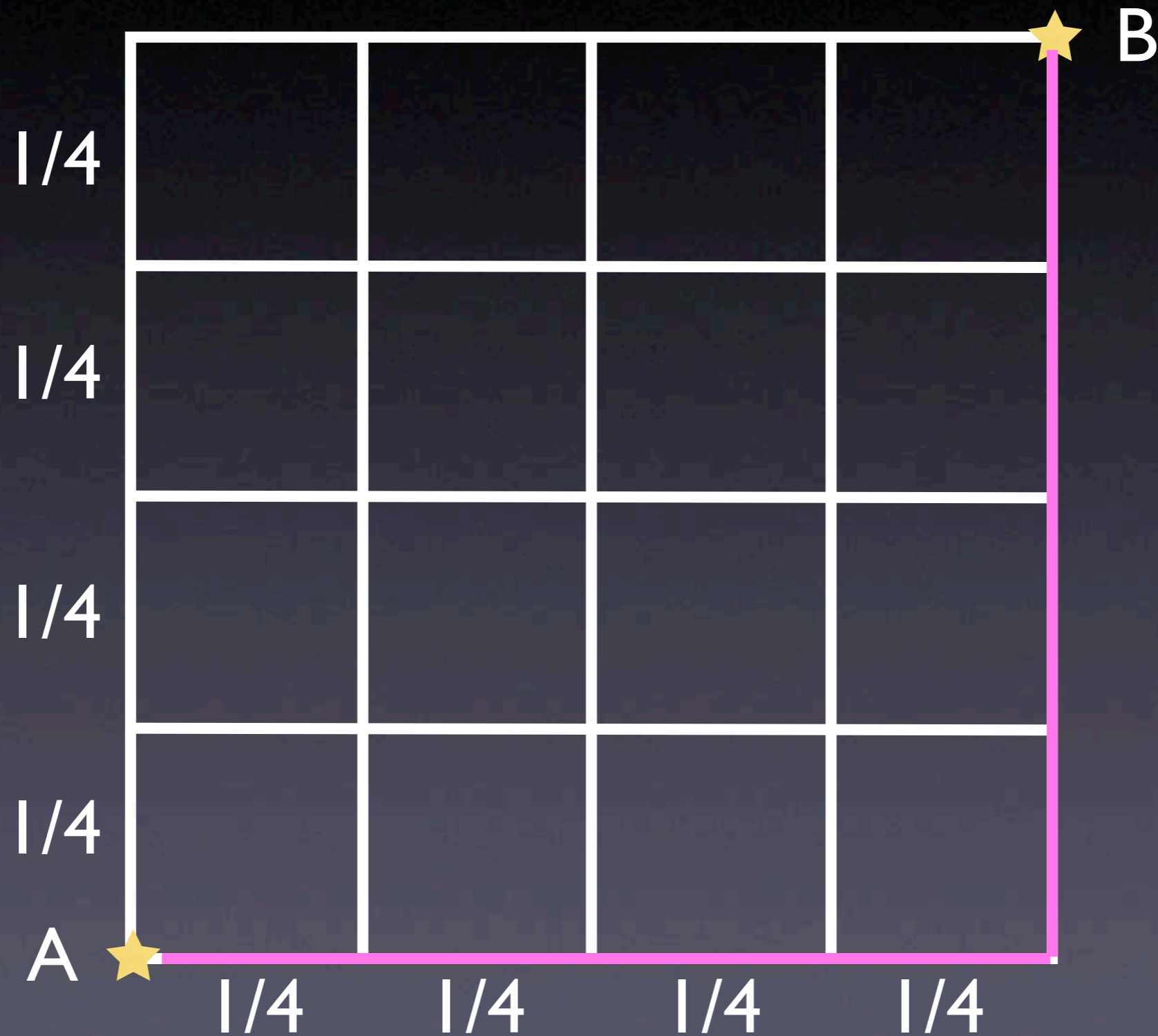
- But the “taxicab distance” from A to B is still exactly 2:



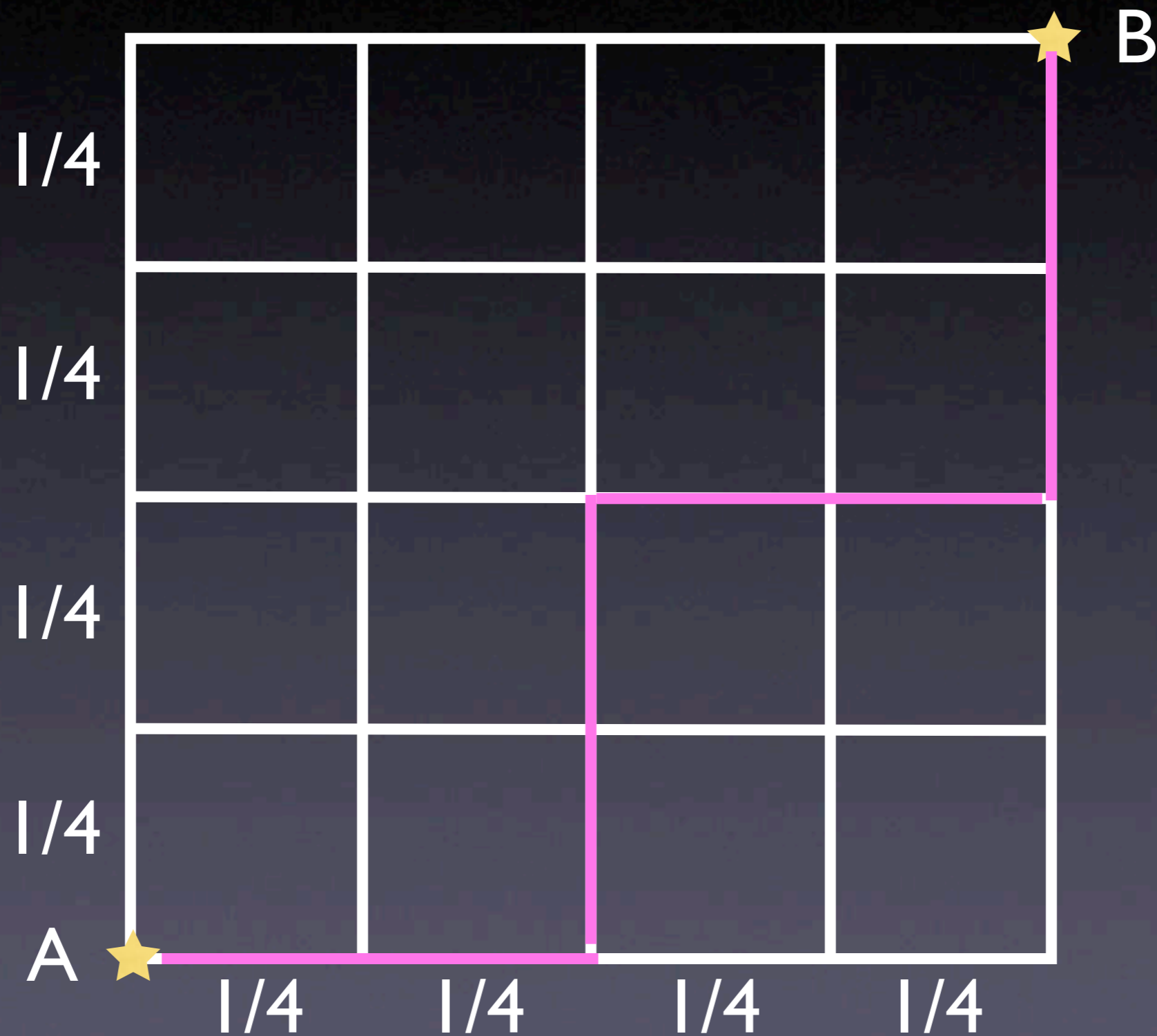
- Suppose we “subdivide” more:



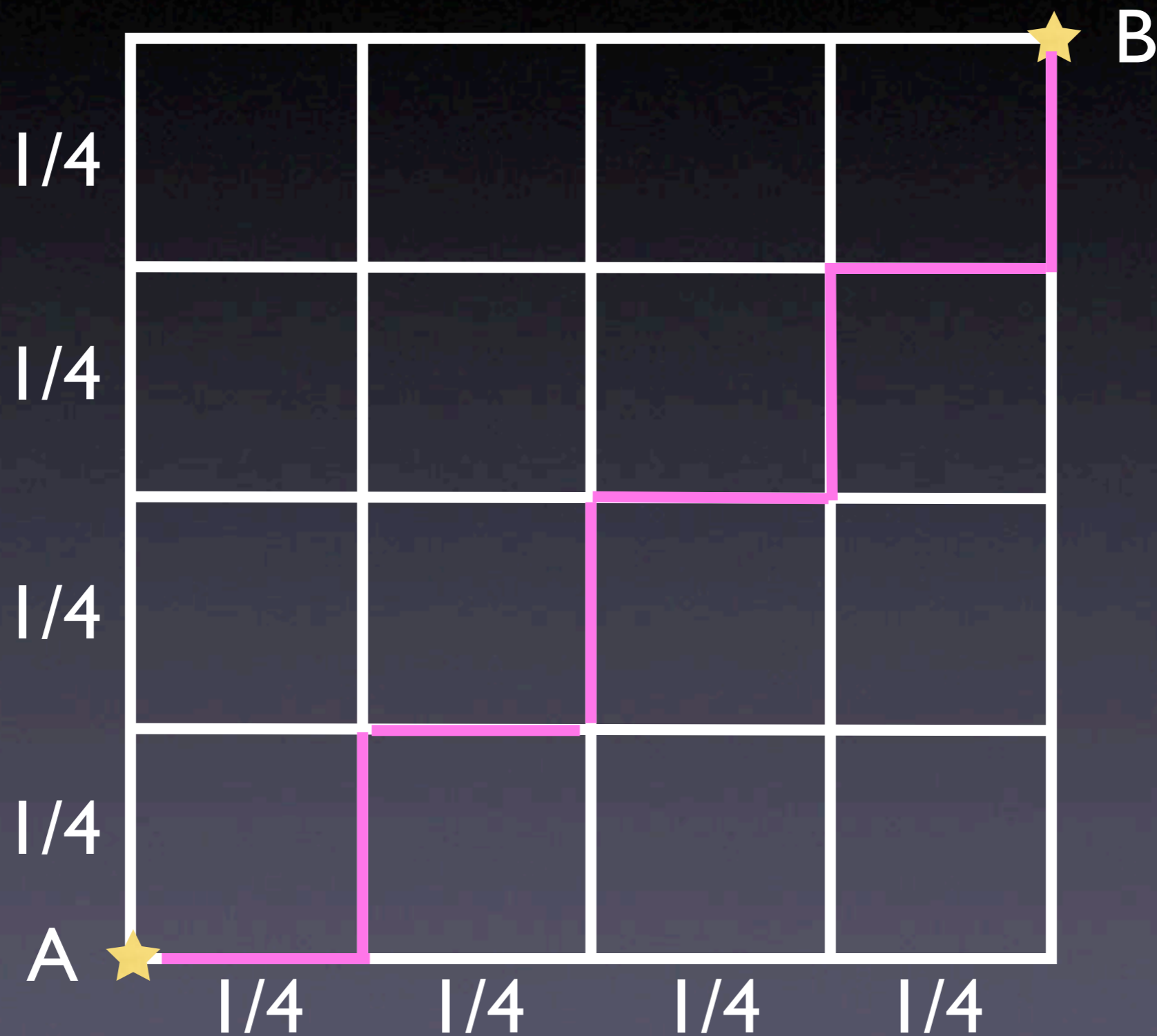
- Again, there are various ways we can go from A to B:



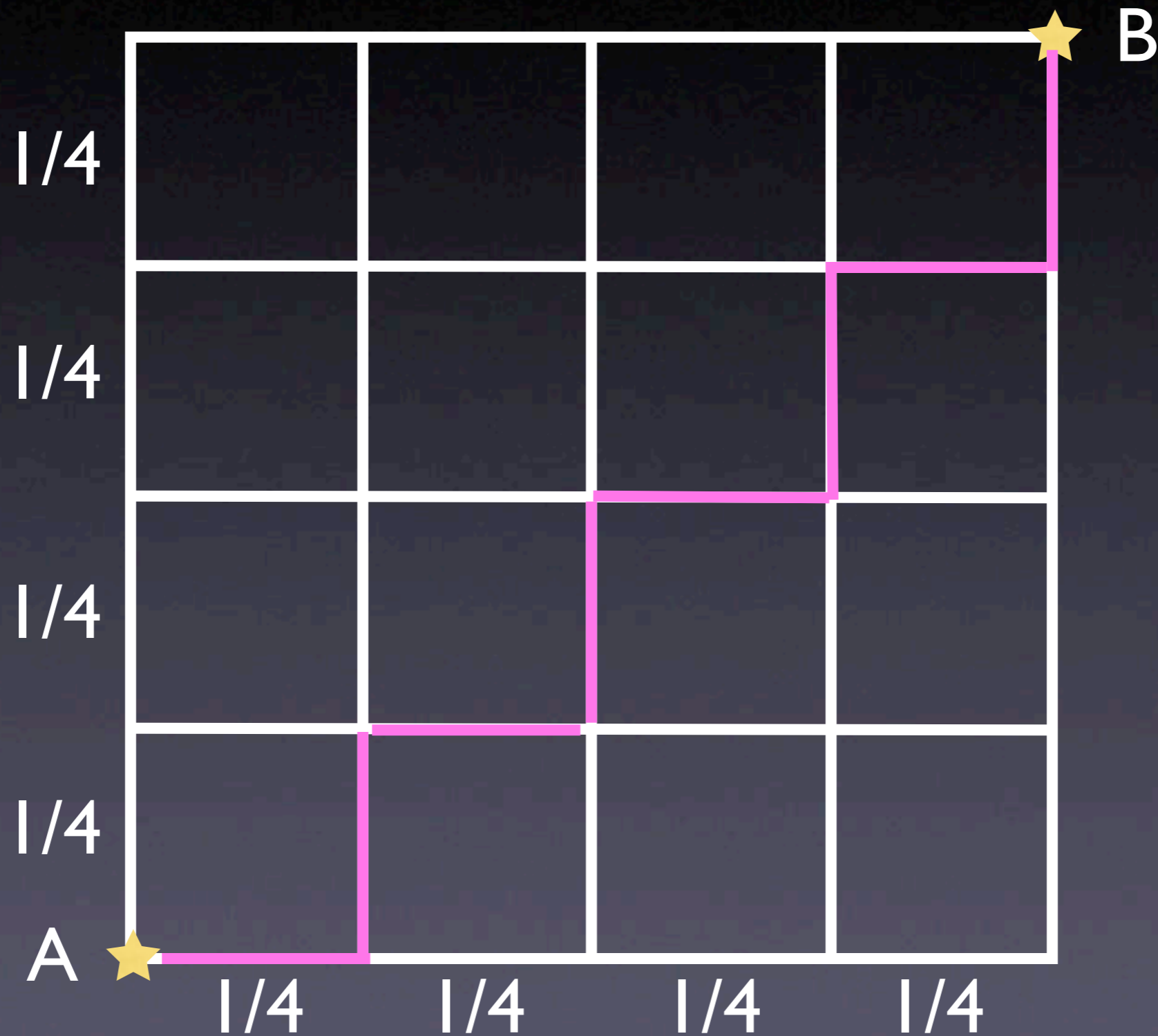
- Again, there are various ways we can go from A to B:



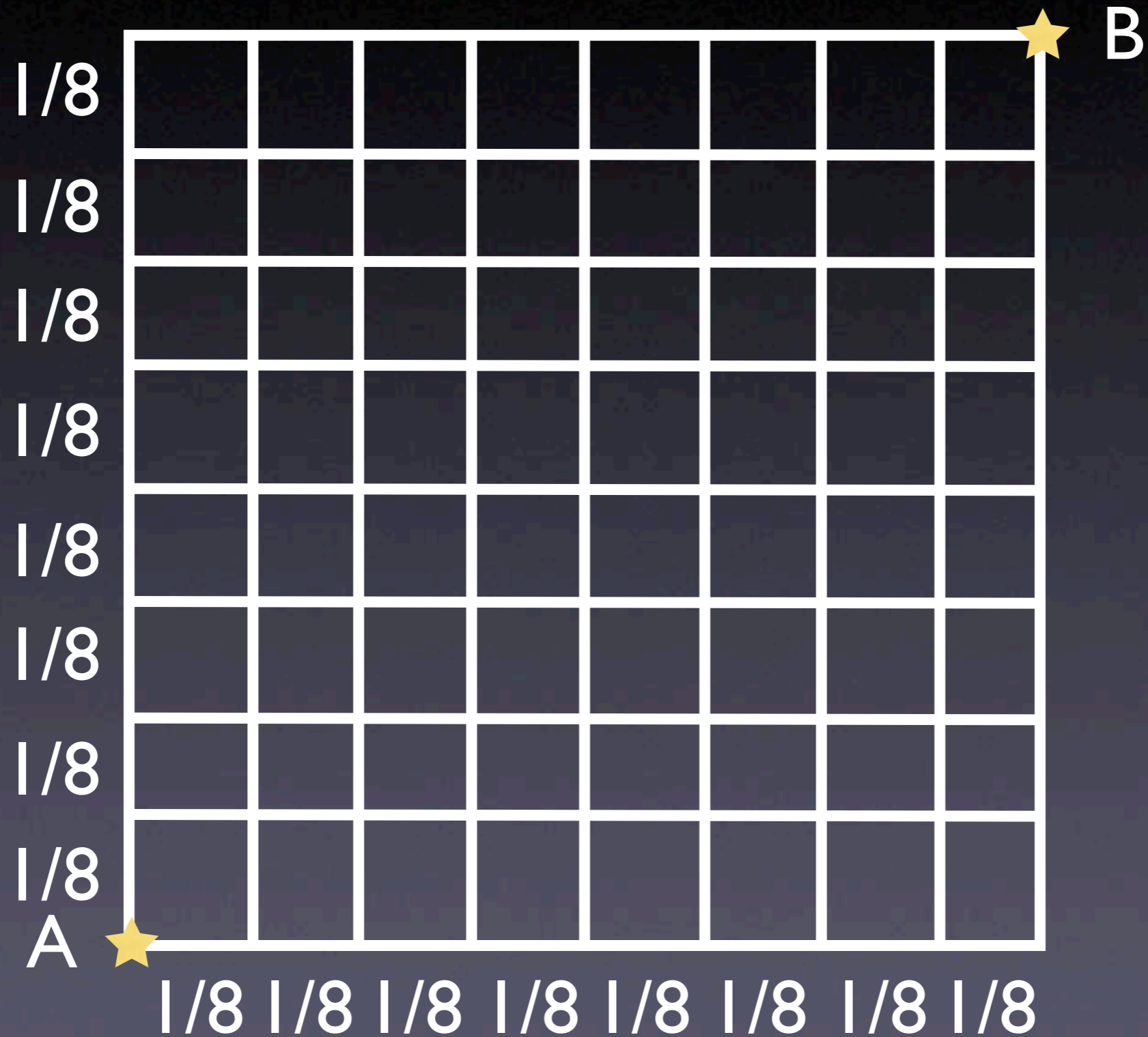
- Again, there are various ways we can go from A to B:



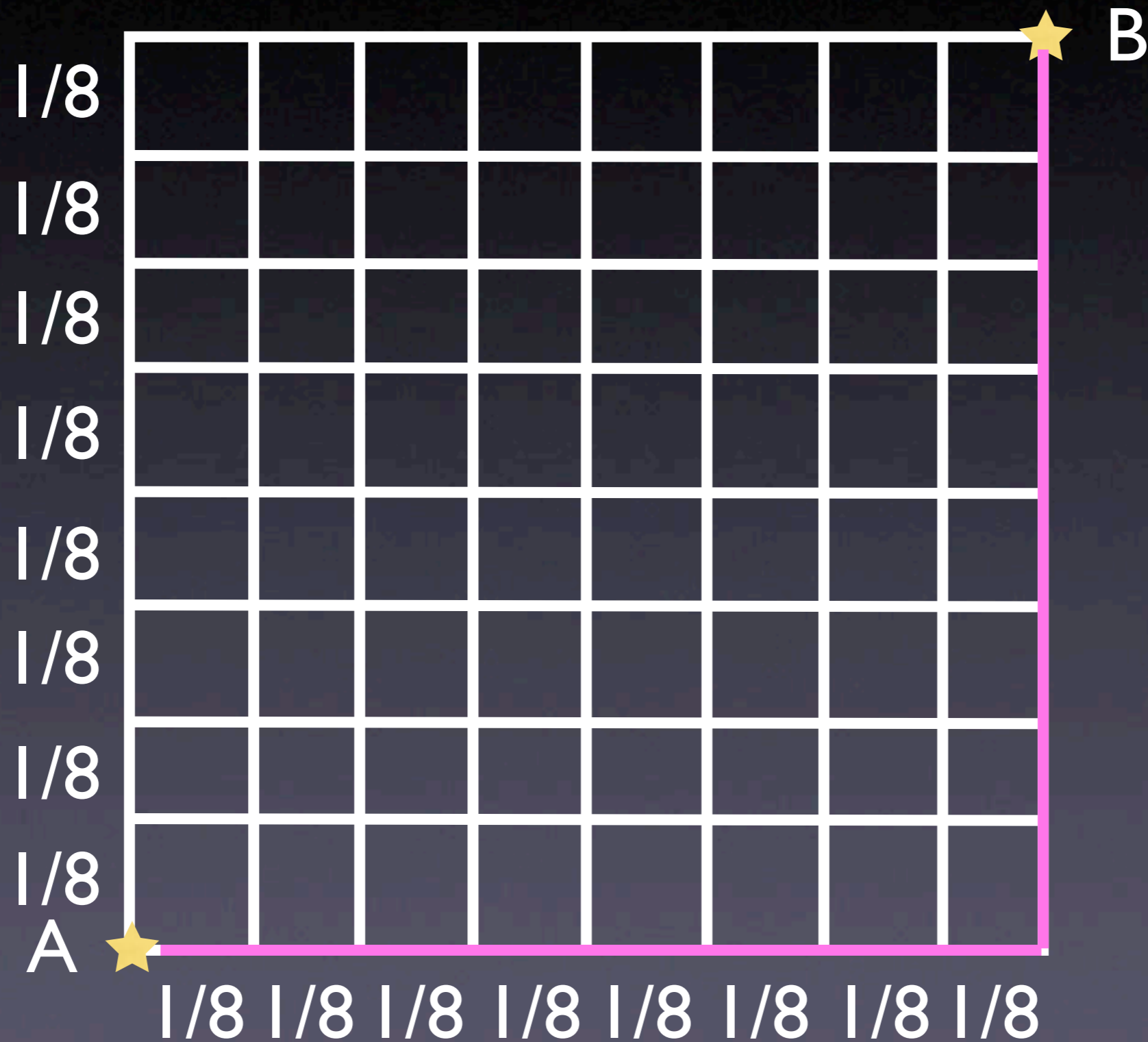
- But whichever way we go, the “taxicab distance” from A to B is still 2.



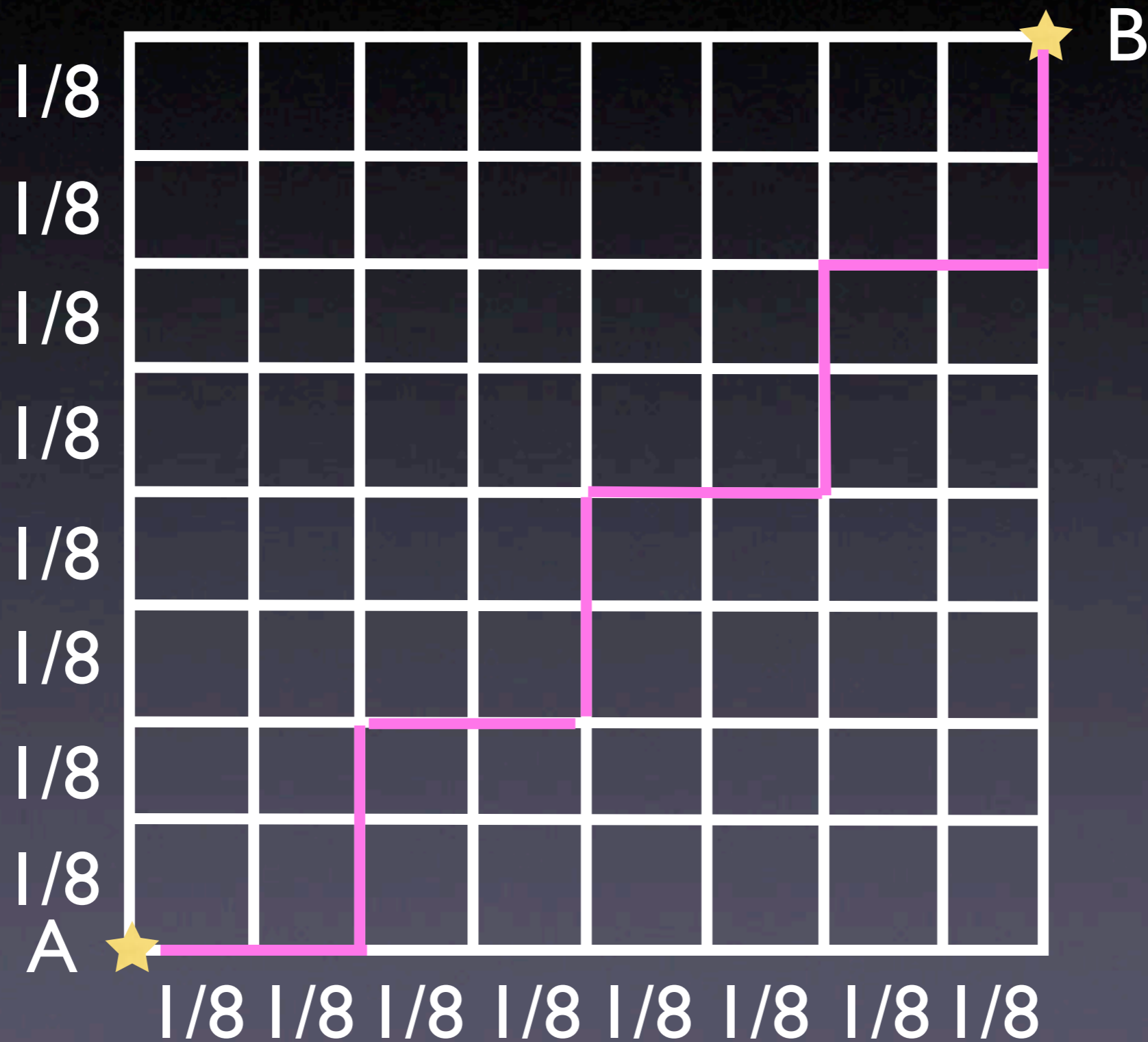
- We could subdivide even more:



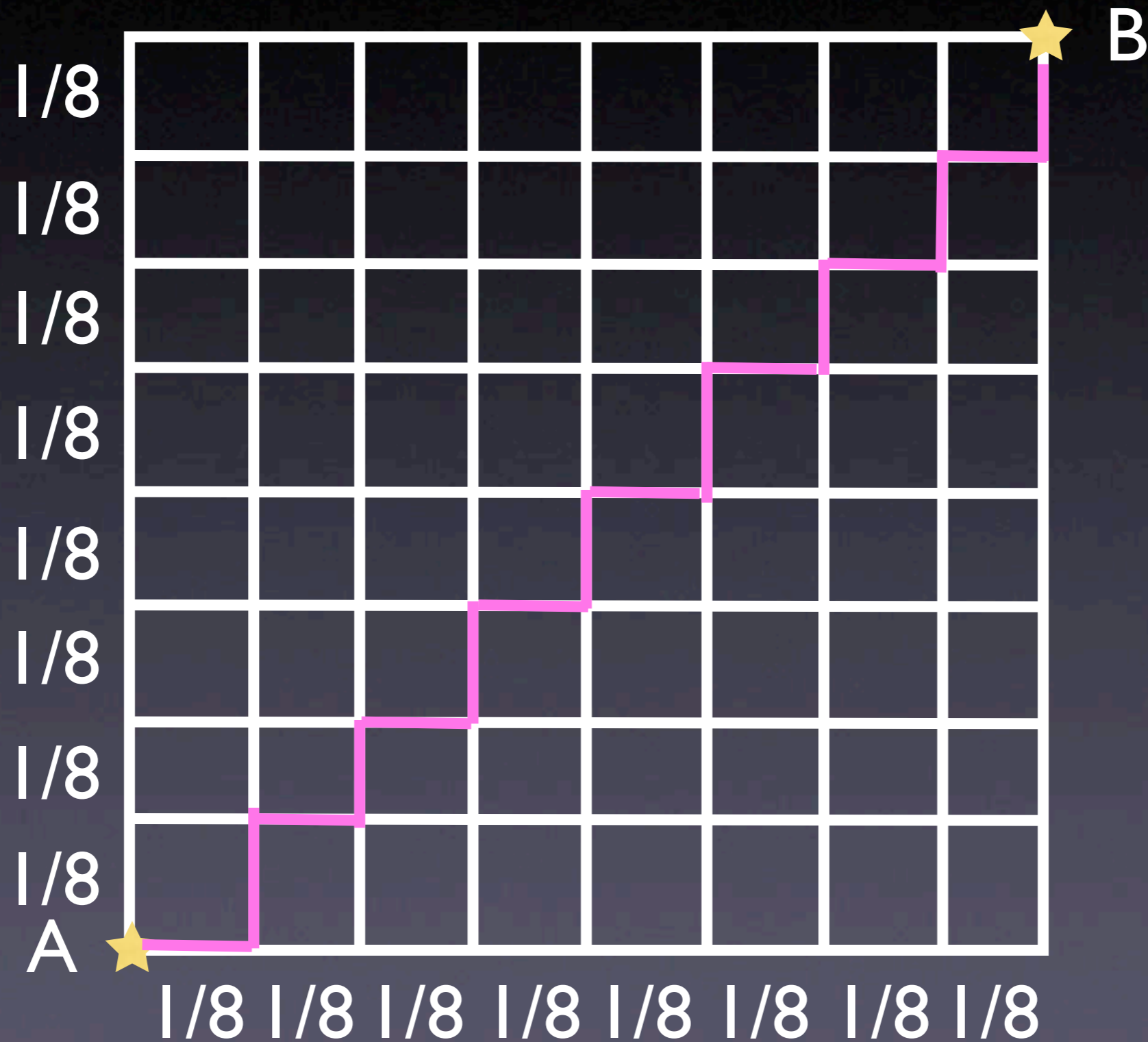
- We can go from A to B in various ways:



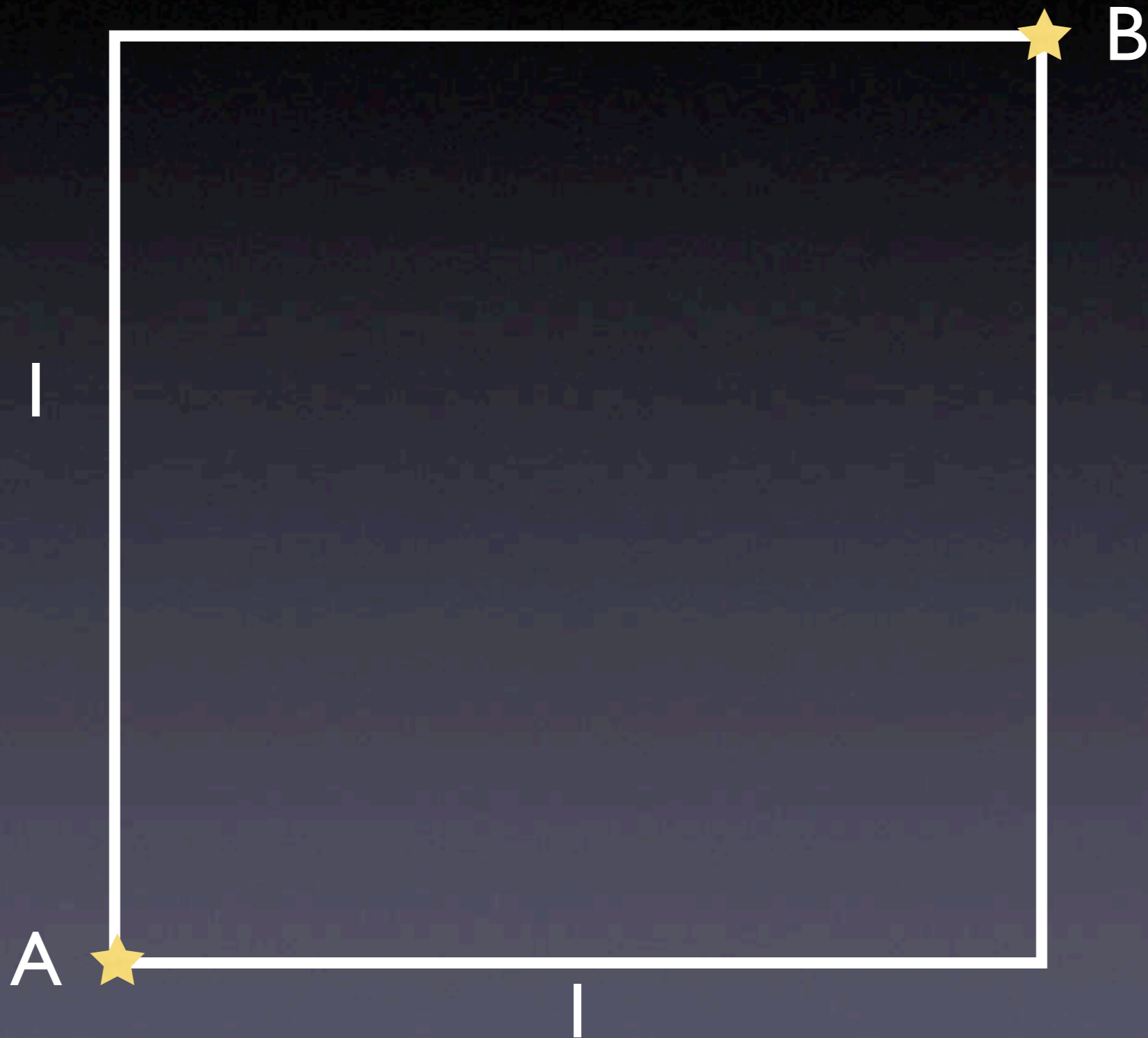
- We can go from A to B in various ways:



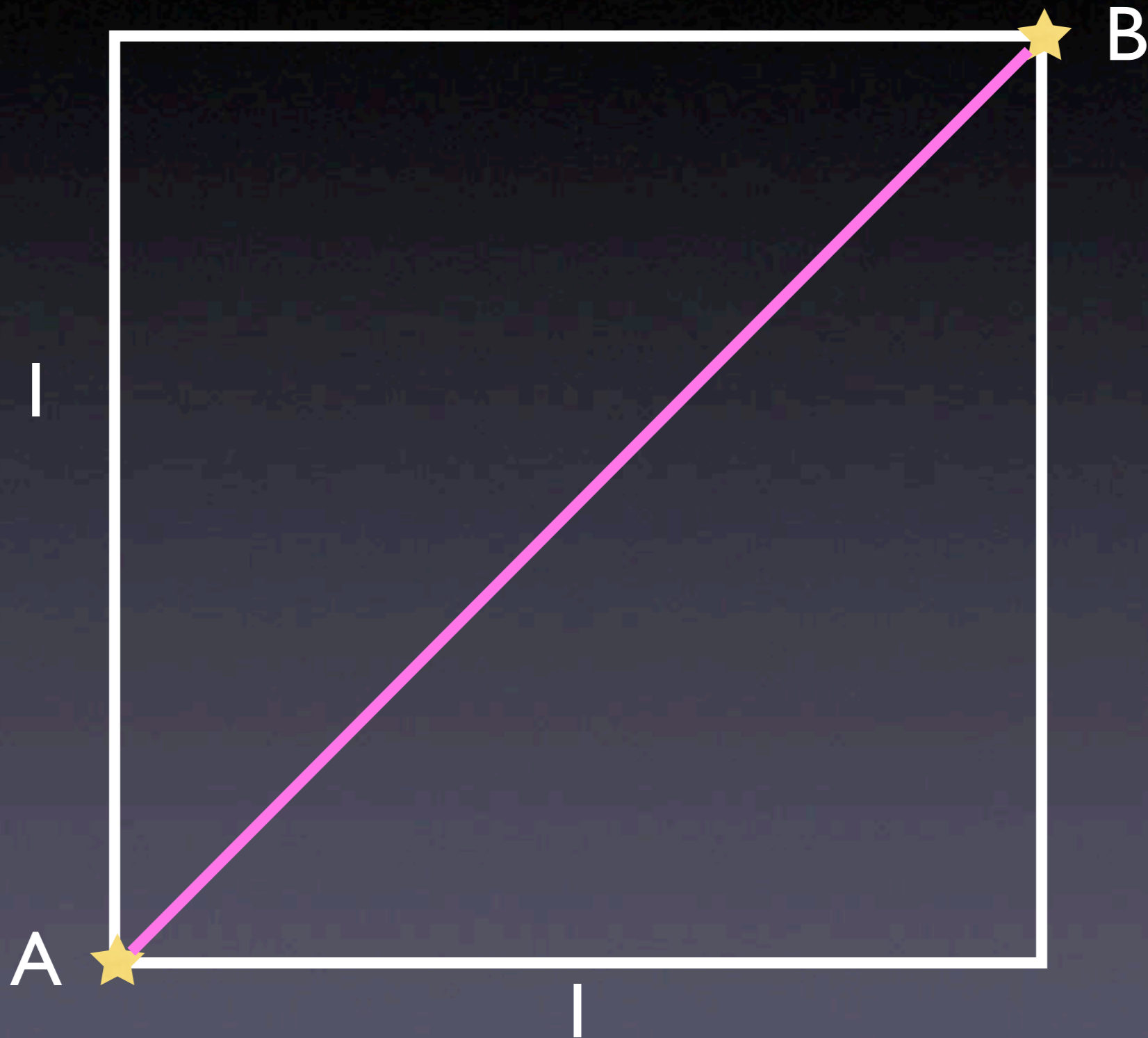
- But whichever way we go, the distance from A to B is still 2.



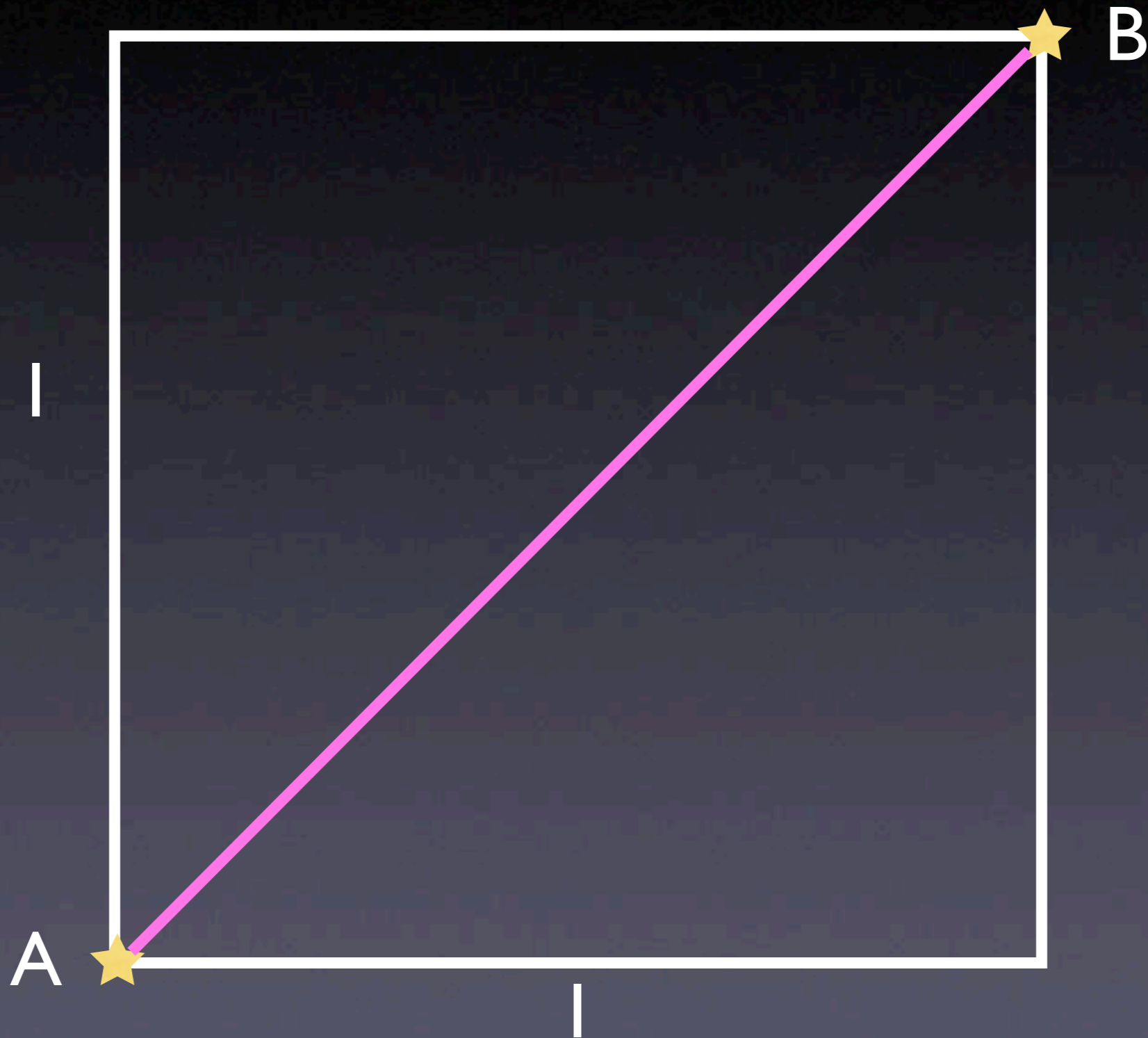
- So now, if we “go to the limit” of the “taxicab geometry” ...



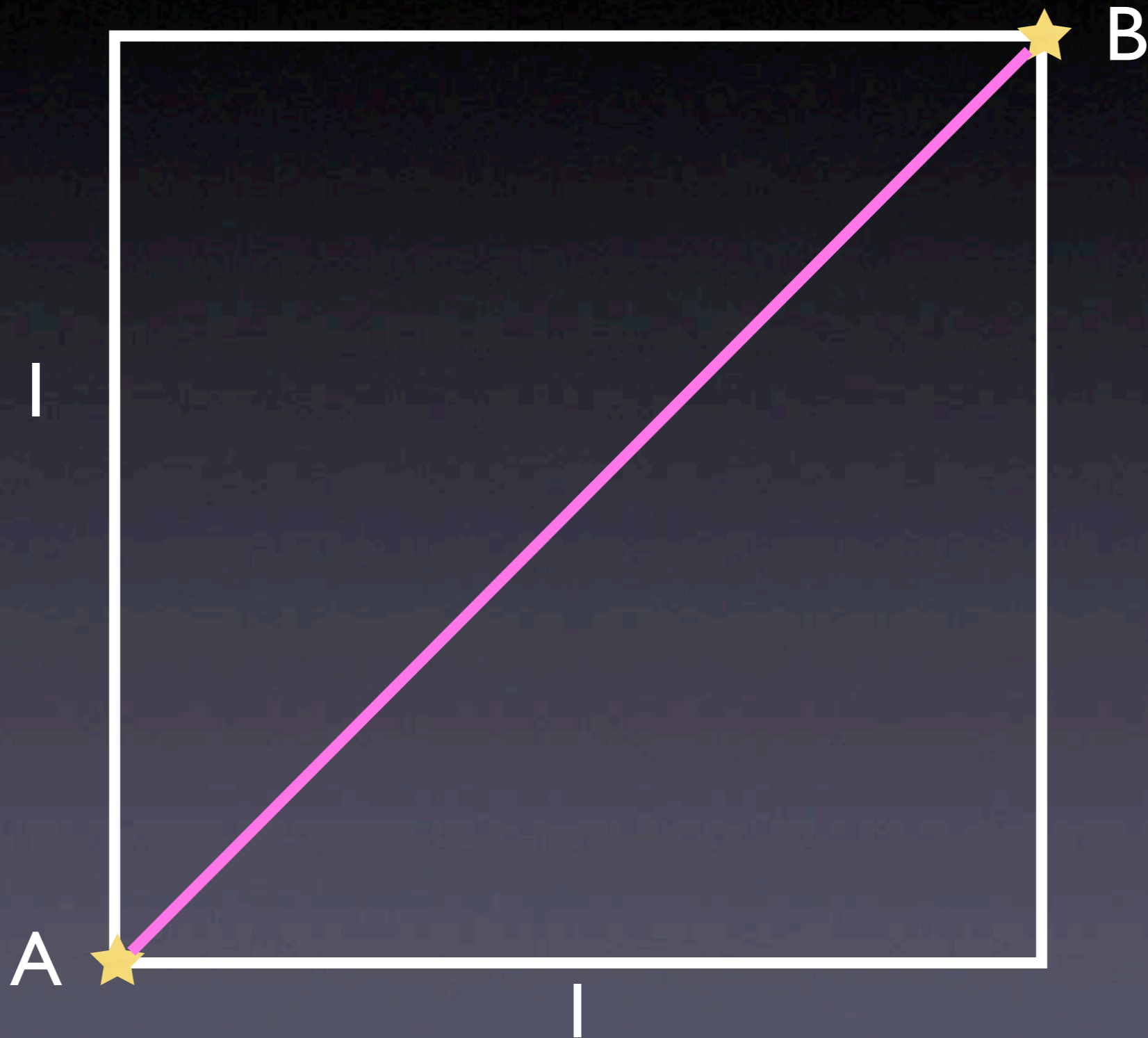
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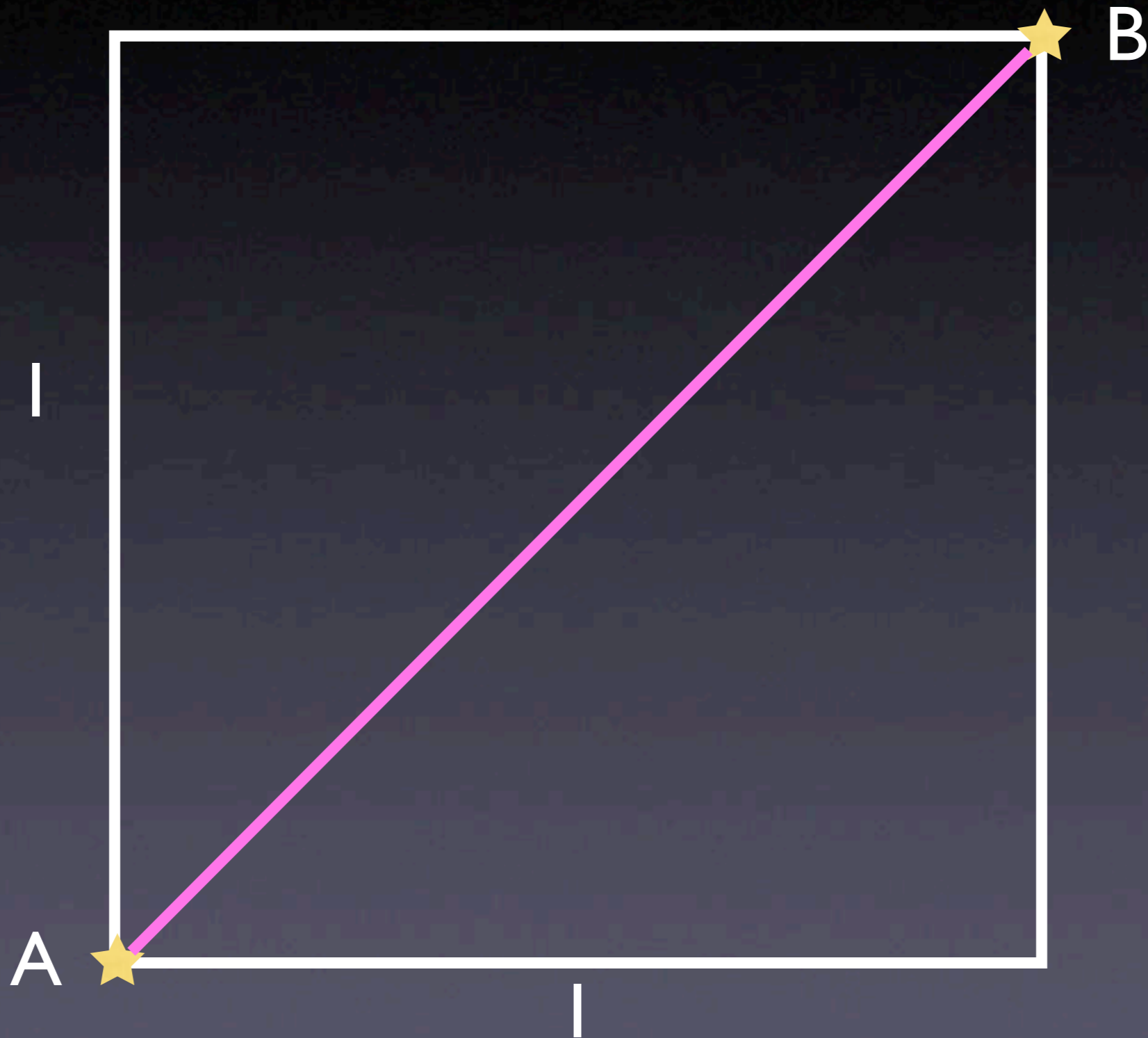
- We can see that the “distance” from A to B is the limit (as n goes to infinity) of 2,



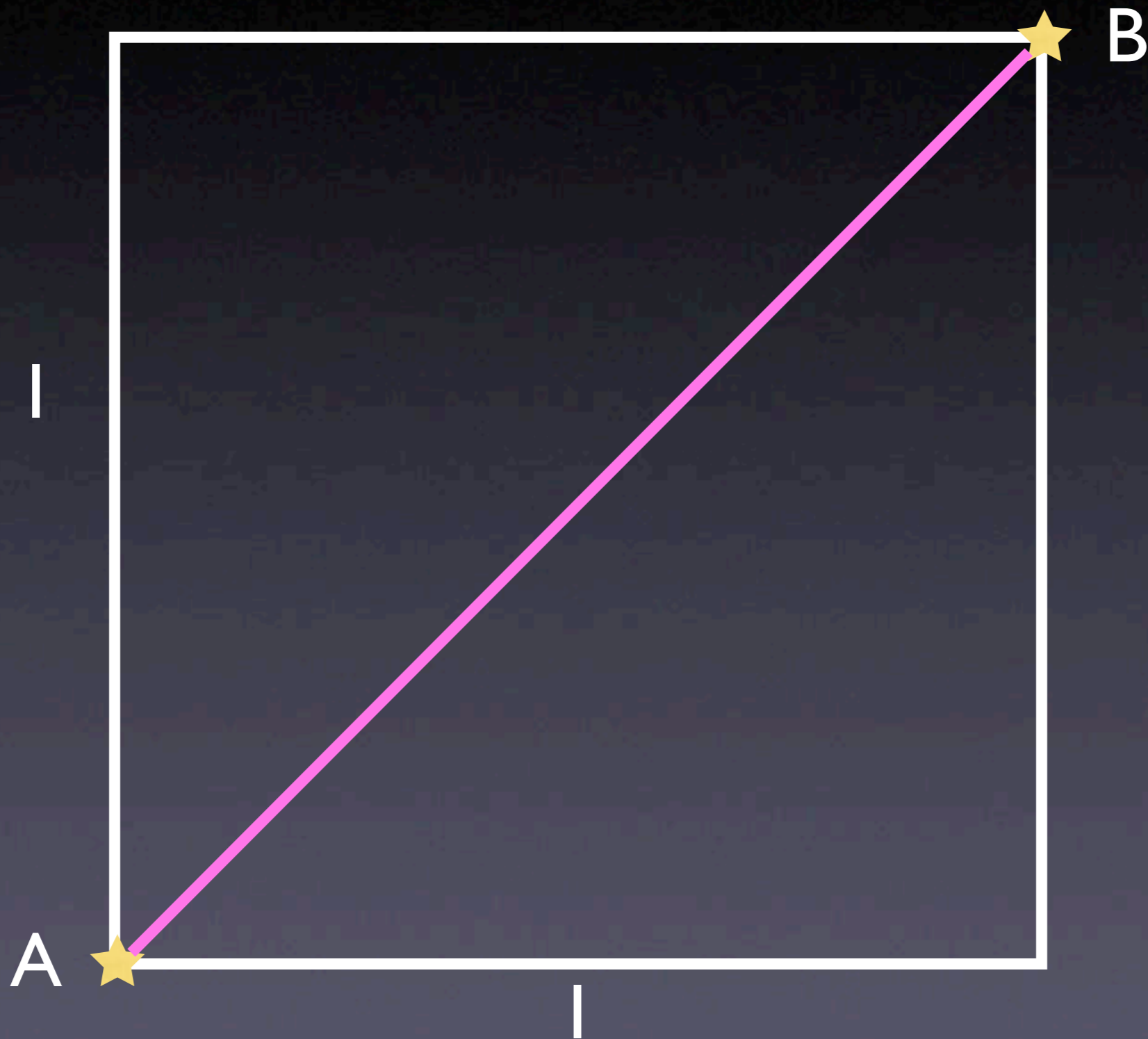
- And so, clearly, the distance from A to B is exactly $2 \dots$



- And thus, since “the limit of the discrete is the continuous,” we must have shown that



- 2 is equal to the square root of 2 !!!



- Exercise: make sense of this!

Fin

