Making Sense

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Making Sense

Introduction / theme / structure ←

- This is a brief introduction to some thoughts about making sense of the world. We all make sense of the world in many ways all the time, but we don't always do it consciously, nor do we necessarily have rigorous / structured approaches to the project . . .

  My goal here is to talk some about how I go about building theories, models and simulations that I can use to make the world make more sense to me – and also that I can use as parts of explanations to help others see the world in potentially more useful ways.

  As a teacher/researcher, I'm always looking for more illumination, and better ways to reveal that illumination.

- One important point for me is that in general in these sorts of projects, I am much more interested in epistemology than in ontology.

  Just briefly, what are ontology and epistemology? I tend to think about them this way:

  Ontology: “What is there?” (i.e., questions of “being”)

  Epistemology: “How do we know?” (i.e., questions of “knowledge”)

  In many respects, I see these as two of the main (or the main two) branches of philosophy, and especially of philosophy of science. They tend to have convergences and divergences, but they drive much philosphizing (and much argumentation over angels and heads of pins . . . :-)

- Ontological questions tend to litter the fields of science: What are the fundamental elements? Are there just four of them? What is the Fifth Element? Is it the quintessence, or just a so-so sci-fi movie? Does caloric exist? Does phlogiston exist? Are there atoms, or not? Do electrons exist? Does the force of gravity exist? What is a gene? What is a species? What is life? Does Truth exist?

  For the most part, I think these questions are largely irrelevant. In many respects, I think they are very often the wrong kinds of questions for scientists to ask . . .

- Epistemological questions also abound: What can we know? How can we best go a trying to learn (gain knowledge) about the world? What role does evidence play in understanding systems? How meaningful is deduction within axiomatic contexts?

Language and meaning ←

Hmmm . . . This is a placeholder for some things I want to write about, but haven't yet.

- Reference and Platonism
- Use and Wittgenstein
- What is meaning, and how does it happen?

Language and meaning - exercises ←

1. Explain why "meaning is use" is meaningful.

Theories, models and simulation ←

- What is a theory? This turns out (at least in a socio-cultural sense) to be a rather difficult question.

  One fairly traditional notion is to use ideas from mathematics. In this form, a *theory* is a collection of axioms, definitions, rules of inference, symbols, "objects," relations, etc. (e.g., the theory of Euclidean Geometry). One can (in theory :-) derive results within the theory, and engage in a general hypothetico-deductive cycle. One "makes a hypothesis" within the framework of the theory, and then checks to see if the hypothesis is derivable within the theory.

  A fundamental question one can ask at this stage is whether the theory is consistent – i.e., whether the collection of axioms, etc., is logically consistent.

- A straightforward example of this is the mathematical theory of *Groups*. A *group* is a quadruple $(G, \cdot, ^{-1}, e)$, where G is a set (of group elements), $\cdot : G \times G \to G$ is a binary operation, $^{-1} : G \to G$ is a unary operation (the inverse in G), and $e \in G$ is a distinguished element of G (the identity element) satisfying the axioms, for $g, g_1, g_2, g_3 \in G$:

    0. $g_1 \cdot g_2 \in G$ and $g^{-1} \in G$ (closure under $\cdot$ and $^{-1}$)

    1. $(g_1 \cdot g_2) \cdot g_3 = g_1 \cdot (g_2 \cdot g_3)$ (associativity)

    2. $e \cdot g = g \cdot e = g$ (identity element)

    3. $g \cdot g^{-1} = g^{-1} \cdot g = e$ (inverses)

(Note that we don't really need axiom 0, since we require that $\cdot$ and $^{-1}$ be a binary and unary operation (respectively) on G, but it is traditional to emphasize the closure property(s)).

Given these axioms, we can prove various things. For example, the identity element e is unique: If $a \cdot g = g$ for all $g \in G$, then $a = a \cdot e = e$ (axiom 2, and then the property of a).

Similarly, inverses are unique: if $g \cdot a = a \cdot g = e$, then $a = a \cdot e = a \cdot (g \cdot g^{-1}) = (a \cdot g) \cdot g^{-1} = e \cdot g^{-1} = g^{-1}$.

- The next step, then, would be to try to construct an *interpretation* of the theory (what might also be called a *model*). In general, this would be a mapping from "object" and “relation” (etc.) symbols in the theory to specific “objects,” “relations,” etc., external to the theory. This process can sometimes be relatively straightforward, and other times remarkably problematic . . .

For example, it is easy to check that the integers form a *group* under addition (i.e., the quadruple $(\mathbb{Z}, +, -, 0)$ satisfies the group axioms). We then have the nice property that any theorem we have proven using the *group axioms* is automatically satisfied by the integers.

Similarly we can develop notions of *symmetry groups* and *crystallographic groups*, with a variety of interesting applications . . .

- An observation we can make here is that the existence of a *model* of a given *theory* assures us that the *theory* is consistent. But a somewhat more subtle question is the *Truth* question. Does it make sense to ask whether *Group Theory* is *True*? I have to say, I can't really make sense of that question. It certainly is the case that *Group Theory* is consistent (it has models), and it is certainly very useful in many contexts, but *True*? I don't know . . .

- Perhaps more interesting are attempts to construct interpretations to real-world objects and phenomena. Unfortunately, this can be quite subtle – in particular, difficulty typically arises in confirming that the axioms actually hold in the interpretation.

  We can now enter into another level of hypothetico-deductive cycle. We make observations in the realm "external" to the "theory," then "turn the crank" to get "predictions," make more observations, and see if the results "match" (e.g., we "do experiments").

  At this point, we are likely to have to make some sense of what we have seen, and decide what to do next.

- Perhaps it would be worth going through a specific example, to see some of the issues.

  Imagine for a moment that it is around 1300 C.E., and you work for the Grand Vizier. He (and the King) believe that the planets affect one's life, and they demand that you "cast the King's horoscope." In other words, you are to describe (in some detail) what the sky would look like at a particular time and place (perhaps some 35 years prior).

  Your first "observation" is that most of the points of light (stars) in the night sky are "fixed." Of course, even this much requires a significant degree of abstraction – if you hold your gaze fixed with respect to the ground you stand on, the stars will "move" – they will "rotate" during the night. The stars are "fixed" with respect to each other. Notice that there is also a (covert) assumption that the stars I see tonight are the same stars I saw last night. I can have my graduate students (apprentices :-) draw maps of the (relative) positions of the stars on successive nights, noting the strong similarities between the maps, I can then make the (simplifying) assumption that they are the same stars. Part of what I am pointing out here is that in building a “theory” there are innumerable background (often unspoken) assumptions necessarily underlying the “theory.” It is probably worth noting that if the “theory” “doesn’t work,” it may be (is?) a nontrivial exercise to figure out which of the explicit and/or implicit assumptions might be changed to get the “theory” to “work” . . .

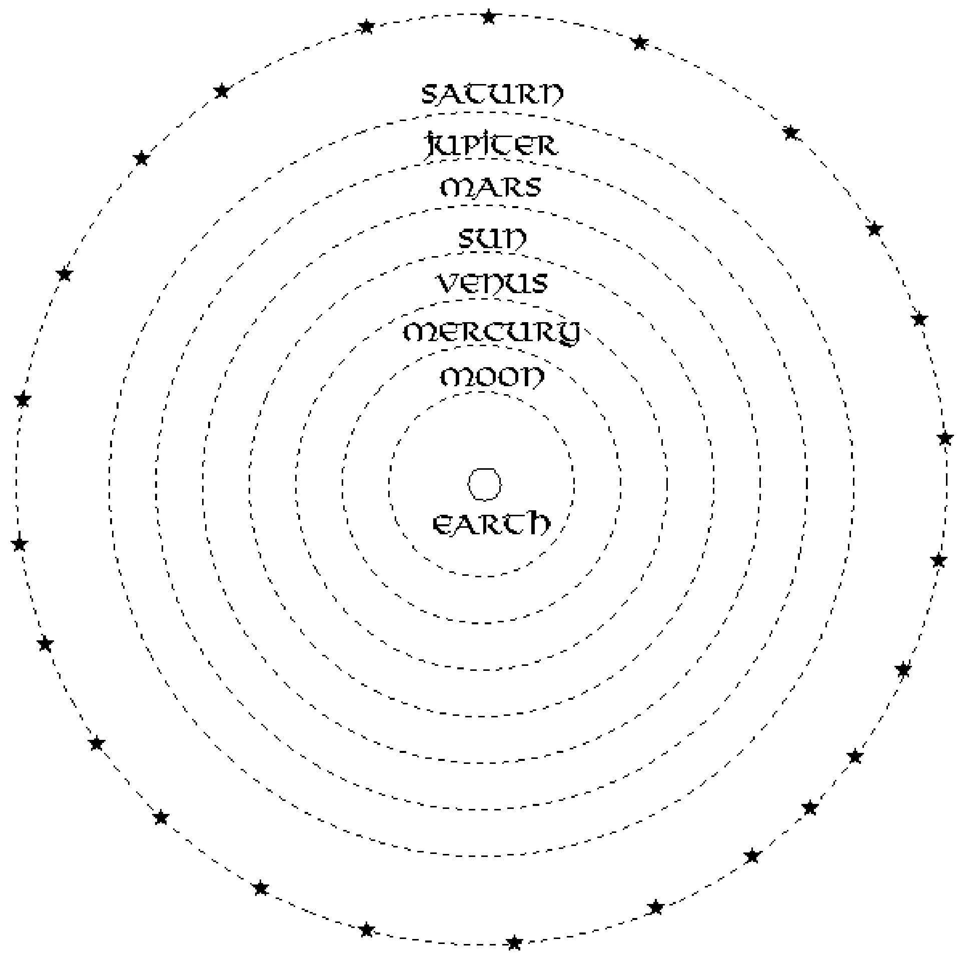
We can now “observe” (with various caveats . . .) that the planets “move” with respect to the fixed stars. Making various assumptions about regularity, continuity, and simplicity in general, we want to build a “model” (an orrery?) of the (relative) motions of the planets.

What are the constraints within which we will build our “model?” A first major constraint is that the planets, being celestial objects, will move in “perfect” ways, and since the circle is the most perfect of shapes, they will move along circular paths. Thus we start building our theory.

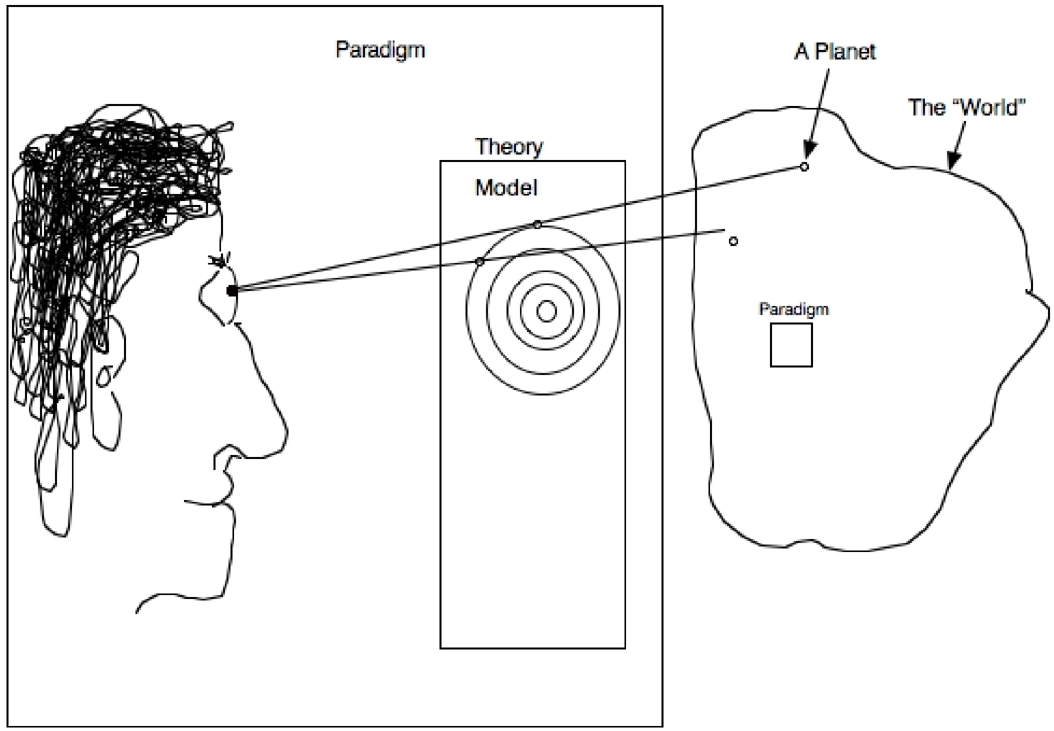
Our theory: The fixed stars are on an encompassing immense sphere. The planets move on circular paths within the sphere. The planets move continuously, smoothly, and at a constant rate along their paths. Each (circular) path has a fixed center and a fixed radius.

We now build a specific model. For each planet, we choose (determine) a specific center, radius, and rate of travel. Without much thought, the circles are all coplanar.

Here is a first picture:



Or, perhaps a better way to think about it:

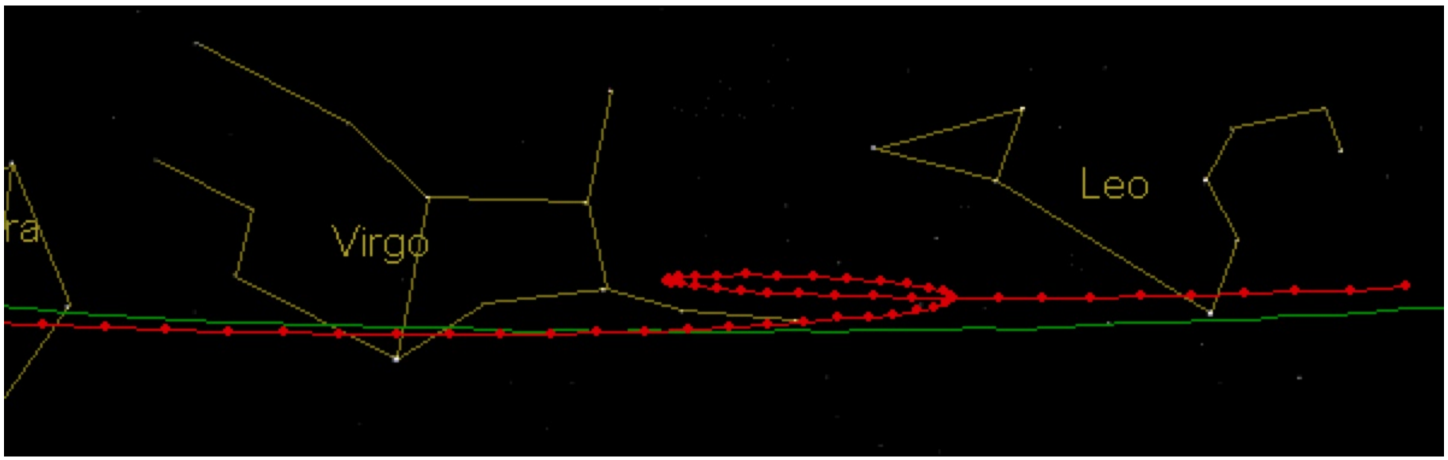


Here the observer (us) looks at the world through the theory/model (from within a perhaps unacknowledged paradigm). The theory/model becomes a lens through which the world is viewed. This lens serves to select/emphasize certain aspects of the world.

Notice that the "prediction" of the model does not exactly match the world . . .

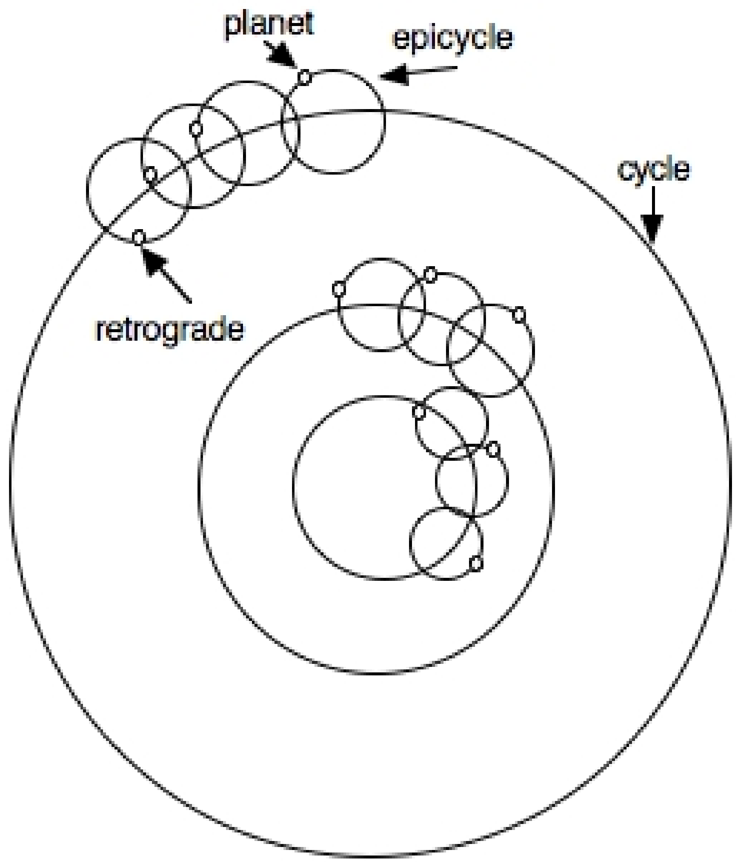
Now what do we do? Given that the "prediction" of our model does not exactly match the world, we have several choices. First, we could declare the match "good enough," take the money from the Grand Vizier, and go on our way. Second, we could modify our model, by changing various parameters (radii, centers, rate of motion, etc.). We could then check each of these revised models to see if one was "good enough."

We might notice that some particular aspects of the phenomena are not consistent with the model. For example, we might notice that the motion of Mars is sometimes *retrograde*:



If none of the revised models was "good enough," we might replace our theory with a different one. In this case (remaining in the same "paradigm") we might "allow" our planets to move in "circles upon circles" (i.e., epicycles), so that sometimes the planet would move "backward."

Here's the next:



We now have lots of parameters we can adjust. We could also add epicycles upon epicycles upon . . . (and, by Fourier, make things match pretty much as well as we want . . .).

- On the other hand, we could even go so far as to step to a new paradigm, and allow our theory to include the earth as one of the moving bodies (no longer distinguishing between celestial and terrestrial), and even allow the paths to be other conic sections, such as ellipses. We might make an orrery like this, with an actual physical crank we can turn:

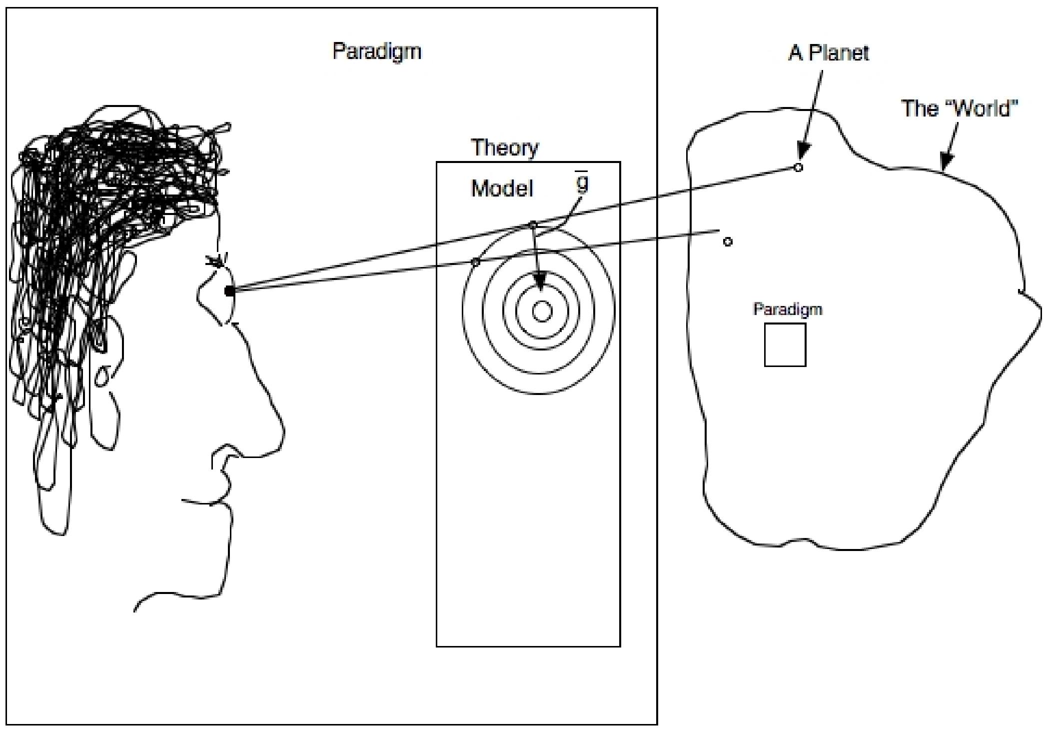
- Hmmm. Are we done now? What do we want a theory/model to do for us? It seems to me in various contexts we would like it to

  1. Give us a good description of the system
  2. Allow us to predict behavior of the system
  3. Give us an explanation of the system
  4. Allow us to control the system

  A nice heliocentric theory/model (Keplerian, say) can do a very good job of describing the solar system, and can allow us to make good predictions (or retrodictions) of the behavior, but it doesn't give us much in the way of an explanation of the system. In the next step, things get very interesting.

Enter Isaac Newton. In the new paradigm, we allow new entities to exist in our theory/model – forces. In particular, gravitational force.

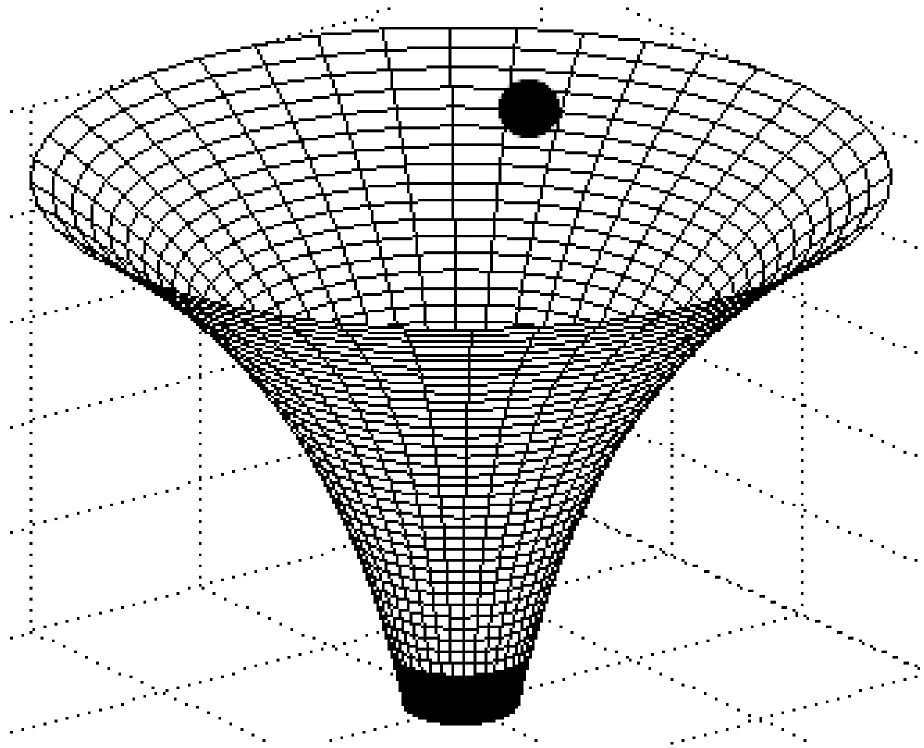
The picture looks very similar to what we had before:



Note, though, that the new entity "gravitational force," shown as an arrow in the model, does not correspond with any observed entity in the world . . .

No one has ever observed the *gravitational force* (i.e., the *force* itself). People have observed things falling, and drawn the conclusion that there must be a *force* that causes the things to fall, but the *force* is not anything anyone has observed – it is an entity in the *model*. The supposed *existence* of the *force* in the world is a conclusion drawn from the model, not through direct observation. Within the paradigm of *forces* and differential equations, etc., it is easy to imagine that we are seeing the *effects of forces*, but it is worth remembering that we are "always already" looking at the world through the lenses of our models, and it is all to easy to mistake aspects of the lens for real phenomena in the world . . .

- One more, to clarify the last observation. In general relativity, an object follows a geodesic within a curved spacetime, with curvature determined by local masses.



Note that there is no “force of gravity” in this model . . .

- On building models:

  My general policy when building a model is to start with the absolute minimum to get the model off the ground. In order to do this, I have to observe / think about the system long and hard, thinking about what might possibly be irrelevant to the aspects of the system's behavior in which I am interested. (See, for example, my "economics" models . . .)

  There must be more to come, but I'll stop writing here for now, and talk instead :-)

Theories, models and simulation - exercises ←

1. What sorts of relations can there be between models and reality?

←

References

[1] Fox Keller, Evelyn, *Making Sense of Life*, Harvard University Press, Cambridge, 2002.

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