Intelligent Patterning

or

Why I've been doing computer science

Brief overview of where I'm headed:

- General problem solving
- Pattern recognition
- Symbols and signs
- Intelligent patterning
- Some history
- What's wrong in computing today
- The intelligent mathematical assistant

General problem solving

- Understanding the problem

  1. Problem context and statement of the problem

  2. Solving the right problem (ill-posed and ill-conditioned problems)

  3. Preconceptions

  4. Language and restating the problem

- The role of experience

  1. Similar problems and analogy

  2. Appropriate tools

  3. Specific experience

- Three basic methods

  1. Plug and grind

  2. Guess and prove

  3. Look it up

- Hypothesis generation and testing

  1. Flexibility and freedom — willingness to try and fail

  2. Recognizing blind alleys, and the value of exploring

  3. Appropriate hypotheses

  4. Lateral thinking

- Recognizing solutions

  1. “A” solution vs. “the” solution

  2. Useful solutions

  3. When a “solution” solves an un-posed, but more significant problem

Pattern recognition

- Images ("visual patterns") vs. "syntactic" patterns

- Symbols as patterns, and symbols as pattern labels

- Patterns of symbols

- Hierarchies of patterns, and symbols as tools for recognizing patterns

- Pattern manipulation

- Learning to recognize patterns, and pattern
  recognition as learning

Pattern recognition examples

- What number comes next in the sequence?
  1, 1, 2, 3, 5, 8, 13, . . .

- What number comes next in the sequence?
  8, 5, 4, 9, 1, 7, 6, 3, . . .

- What letter comes next in the sequence?
  E, T, A, O, I, N, S, H, . . .

- In which row does Z go?
  A, E, F, H, I, K, L, M, N, T, V, W, X, Y
  B, C, D, G, J, O, P, Q, R, S, U

- What letter comes next in the sequence?
  W, L, C, N, I, T, . . .

Symbols and signs

- The utility and power of symbols

- Choosing symbols, naming and pointing

- Symbols as "chunking" tools

- When to use symbols

  1. The importance of anonymity (e.g., the lambda calculus)

  2. Place holders (variables)

  3. Temporary and tentative symbols

- Signs, symbols, content and meaning

Intelligent patterning

- Creativity and Art
  1. Knowing when to pattern
  2. Symbol attachment and creation; patterns/symbols as revealers and concealers
  3. Levels of patterning

- Multiple patterns and selection
  $(x-1)(x-2)(x-3)-6$
  $x^3-6x^2+11x-12$
  $(x-4)(x^2-2x+3)$

- Adaptive pattern recognition

- Are the patterns really there?

Some history

- Physics
- Philosophy (theory of knowledge)
- Mathematics
  1. Matrix manipulation
  2. Topology
  3. Algebra
  4. Lie groups
  5. Manifolds and relativity theory
  6. Algebraic topology

We have the map $b_n : \Sigma^2 U(n) \to SU(n+1)$ given by

$$b_n(g,r,s) = [i(g), v_n(r,s)]$$

where $i(g)$ is the inclusion, $[g,h] = ghg^{-1}h^{-1}$ and

$$v_n(r,s) =$$

$$\begin{bmatrix} \alpha & 0 & 0 & \cdots & 0 & \beta(-\overline{\alpha})^0 \\ \beta(-\overline{\alpha})^0\overline{\beta} & \alpha & 0 & \cdots & 0 & \beta(-\overline{\alpha})^1 \\ \beta(-\overline{\alpha})^1\overline{\beta} & \beta(-\overline{\alpha})^0\overline{\beta} & \alpha & \cdots & 0 & \beta(-\overline{\alpha})^2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ \beta(-\overline{\alpha})^{n-1}\overline{\beta} & \beta(-\overline{\alpha})^{n-2}\overline{\beta} & \cdots & \cdots & \alpha & \beta(-\overline{\alpha})^n \\ -(-\overline{\alpha})^n\overline{\beta} & -(-\overline{\alpha})^{n-1}\overline{\beta} & \cdots & \cdots & -(-\overline{\alpha})^0\overline{\beta} & -(-\overline{\alpha})^n \end{bmatrix}$$

where

$$\alpha = \alpha(r,s) = \cos(\pi r) + i\sin(\pi r)\cos(\pi s)$$

$$\beta = \beta(r,s) = i\sin(\pi r)\sin(\pi s)$$

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$ b_n: \Sigma^2U(n) \rightarrow SU(n+1) $ \newline
given by
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where $i(g)$ is the inclusion,
$\left[g, h\right] = ghg^{-1}h^{-1}$ \newline
and

$ v_n(r,s) = $

\[
\left[ \begin{array}{cccccc}
\alpha & 0 & 0 & \cdots & 0 & \beta (-\overline{\alpha})^0 \\
\beta (-\overline{\alpha})^0\overline{\beta} &
   \alpha  &  0  & \cdots & 0 &
   \beta (-\overline{\alpha})^1 \\
\beta (-\overline{\alpha})^1\overline{\beta} &
   \beta (-\overline{\alpha})^0\overline{\beta} &
   \alpha & \cdots & 0 & \beta (-\overline{\alpha})^2 \\
\vdots & \vdots & \vdots &  & \vdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\vdots & \vdots & \vdots &  & \vdots & \vdots \\
\beta (-\overline{\alpha})^{n-1}\overline{\beta} &
   \beta (-\overline{\alpha})^{n-2}\overline{\beta} &
   \cdots &  \cdots & \alpha &
   \beta (-\overline{\alpha})^n \\
-(-\overline{\alpha})^n\overline{\beta} &
   -(-\overline{\alpha})^{n-1}\overline{\beta} &
   \cdots &  \cdots & -(-\overline{\alpha})^0
   \overline{\beta} & -(-\overline{\alpha})^n \\
\end{array} \right]
\]
where
\[ \alpha = \alpha(r,s) =
\cos(\pi r) + i \sin(\pi r)\cos(\pi s) \]
\[ \beta = \beta(r,s) = i \sin(\pi r)\sin(\pi s) \]
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What's wrong in computing today

- Not enough resolution on displays
- Not enough processing power and memory
- Not enough parallelism
- Software tools are "flat" and sequential rather than hierarchical

The intelligent mathematical assistant

- Adaptive symbolic input and output
- Strong basic skills (all of arithmetic through college calculus and elementary discrete structures)
- First order logic capabilities
- Adaptive “patterning” and “symboling”
- Elementary hypothesis generation and testing